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PROSPECTIVE ELEMENTARY SCHOOL TEACHERS' INTERPRETATION OF NON TRADITIONAL-ARITHMETIC OPERATIONS

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Abstract
Although standard operational algorithms provide with efficient written methods, they are incompatible with intuitive approaches. Children are expected to interpret problems in meaningful way and link conceptual and calculational aspect of mathematics in primary education in mathematics program. Through encouraging informal written strategies and mental methods, children develop confidence in their own approaches to problem solving and feel enthusiasm for mathematics. In that sense; teachers should encourage their students to use students' own strategies and accept students' solutions if it is correct. The aims of this study to reveal prospective elementary school teachers' interpretation about nontraditional operational algorithms. The participants was 36 prospective elementary mathematics teachers studying in their third year in the departmant of elementary education in Turkey. In this study, qualitative case study design was used to collect data. The findings of this study analyzed into four headings.

Keywords: mathematics education, prospective elementary school teachers, standard operational algorithms, pedagogical content knowledge.

INTRODUCTION

There has been great deal of discussion about distinction between conceptual and procedurel knowledge and which of them is more important than other in the field of mathematics and science education. Discusion of procedural and conceptual knowledge may felt in the discipline of mathematics because of its highly structured and clearly defined contents. Hiebert and Lefevre (1986) claim that: Conceptual knowledge is characterized as knowledge that is rich in relationships. Its cognitive aspect evaluated as "unit of conceptual knowledge can not be isolated piece of information (p.1), so; it is a part of conceptual knowledge only if the holder recognizes its relationship to other piece of information while procedural knowledge has two parts namely: knowledge of formal language or symbolic representations and knowledge of rules, algorithms, procedures. According to Skemp (1976) many teacher prefer to teach insturumentaly and do not wait their students produce non-standard approach; beacause procedural knowledge has some advantages: Within its own context, instrumental mathematics is usually easier to understand and the rewards are more immediate, more apparent. Skemp (1976) also state that a ralational understanding of mathematics provide individuals with flexibilty solving mathematical problems via giving a number of pathways to connect ideas.

Is there a flexibility and adaptivity in nature of mathematical understanding? According to Wilkerson and Wilensky (2011), "the flexibility and adaptive nature of mathematical undestanding describe the structure of knowledge as a network of relations between different properties, objects and prosedures" (p.24). Papert (1993) also describe learning as a making connections between mental entities that has been alredy exist. Allan (2011), reconceptualize mathematics as a dynamic discipline to be explored and created then a static domain to be mastered without thought or question.

Are standard algorithms for arithmatic operations easy to understand for students? And are they support students conceptual understanding? Research studies have identified difficulties with the standard algorithms which provide efficient written methods when they are understood but often lead to errors where they are incompatible with intuitive approaches (Anghileri, 1998). When faced with large numbers, many children continue to use inefficient counting or tallying strategies (Anghileri, 1999) Children are expected to interpret problems in a meaningful way, making connections between the conceptual and calculational aspects of

mathematics. By focusing on the development of number sense through encouraging mental methods and informal written strategies, children will develop confidence in their own approaches to problem solving and maintain an inclination and enthusiasm for mathematics.

It is argued that teaching standardised procedures for calculation causes 'cognitive passivity' and 'suspended understanding' as they do not correspond to the way people naturally think about numbers. Studies of workplace mathematics show that pencil and paper methods used by adults are rarely those traditionally taught (DES/WO, 1982) and workplace requirements differ from skills taught in school (Harris, 1991). This finding is also supperted by Nunes, Schliemann and Carraher (1993) reseach results. They conducted reseach on mathematical problem solving among Brazilian children who both having formal instruction in school and selling items on the street. Their studies show that children with limited amounts of formal education could be very successful at "Street mathematics "and very unsuccesful at "school mathematics" although the same aritmatic operations were asked. Nunes and Bryant (1996), associate this situation with socioculturel teory which explain learning as community of practice in dynamic process. Forman (2003), explains different success result from school-learning algoritm do not seem to transfer readly to out of school task. The Street mathematics problems were solved orally while the school like problems were solved using written algorthms. We may explain this situation as children use their own strategies in solving the street settings, whereas in school settings they use procedurel algorithm that is thought and required by teachers.

How do teachers evaluate students' non-standart algorithms about operation? Acording to Shulman (1976) teacher knowledge namely, content knowledge, pedegojical content knowledge and curricular knowledge may effect their opinion. Shulman (1986) also said that knowing subject for teaching requires more than knowing its fact and concepts known as content knowledge. He stated (1986) that " the teacher need not only understand that something is so; must further understand why it is so; on what grounds its warrant can be asserted; under what circumstance our belief in its justification can be weekend and even denied"(p.9). Teachers also have strong pedegojical content knowledge such as knowing different representation of ideas, analogies, illustration, examples, explanations and demostrations (Shulman, 1986). Based on definition of Shulman's pedegojical content knowledge Ball, Thamas and Phelps (2008) divide teachers content knowledge into common content knowledge and specialized content knowledge; divide pedegojical content knowledge into knowledge of content and students and knowledge of content and teaching. Ball, Thamas and Phelps (2008) defined teachers' specialized content knowledge as combination of the mathematical knowledge and skill unique to teaching. "Specialized content knowledge requires understanding different interpretations of the operations in ways that studens need not explicitly distinguish; it requires appreciating the difference between "take away" and "comparision" models of subtraction…" (Ball, Thamas and Phelps ;2008, 400). "Knowledge of content and students" is also important domain for techers which gives teachers ability to anticipating what students are likely to think and what they find confusing (Ball, Thamas and Phelps ;2008). Teachers' specialized content knowledge and knowledge of content and students may effect teachers' interpretation of students non-standard algorithms. I drew heavily on Ball, Thamas and Phelps (2008) theorical model of domains of teacher candidates' professional knowledge to develop the knowledge and beliefs components of the framework.

Based on the literature, although standard operational algorithms provide with efficient written methods, they are incompetible with intuitive approaches. Children are expected to interpret problems in meaningful way and link conceptual and calculational aspect of mathematics each others in primary education in mathematics program. Looking computation as process, in which the students are creative and inventive may have some adventages. Through encouraging informal written strategies and mental methods, children may develop confidence in their own approaches to problem solving and feel enthusiasm for mathematics. Students may think that they have a say in their own learning. Their role in mathematics classroom should not be following the procedure to solve the given task but do mathematics actively. In that sense; teachers should encourage their students to use their own strategies and accept their students' solutions if it is correct. Since today's preservice teachers are tomorrow's elemantry mathematics teachers, their beliefs about classroom practices and specialized content knowledge play an important role in shaping their future teaching and implication of mathematics education. As Allan (2011) state "more democratic mathematics classroom begins with teacher preparetion and investment in ongoing professional develeopment for mathematics teachers at every grade." The aims of this is to investigate prospective elementary school teachers' interpretation of non-traditional

arithmetic operations. The importance of this research is that there is not any reserach on prospective teachers' specialized content knowledge and knowledge of content and students about non-traditional arithmatic operations.

METHODOLOGY

Qualitative case study design was used to support methodological perspective and findings of the research study. In this research design, preservice teachers' pedagogical content knowledge was analyzed within the context of elementary mathematics teacher education program.

36 prospective elementary school teachers in an undergraduate teacher education program in İstanbul were selected by using purposive sampling. 32 of 36 students were women while 4 of 36 were men. The sampe purpusely selected from third year elementary education faculty students who had already successfully completed teaching mathematic 1 for elemantary students. One of the aim of this couse is about how to teach arithmetic operations to elementary students. Data were collected at the end of participants' undergraduate course work of mathematic education 2 at the beginning of the spring semester of 2012-2013 academic year .

To gather and triangulate information, participants were fistly administered with the non-traditional arithmatic operation questionare to examine prospective elementary teachers' specialized content knowledge. The non-standard arithmatic operation questionare was taken and adapted from Ball, Thames and Pheps (2008) and Lampert (1986). Necessary revisions and additions were made. More specifically, possible solutions of elemantry students' non-traditional arithmetic operation were given to prospective elemantry teachers and three questions are asked: 1-whether the solutions are corrrect, 2-which strategies do students use to solve these non-traditional aritmatic operations, 3- whether prospective teachers want their students to solve arithmetic operations with these nontraditional methods or standart methods? And why?" These questions measure conceptual understanding, adaptive reasoning of prospective teachers. The non-traditional arithmetic operation questionare was administered to regular course hours, and all the prospective elementary teachers who attended to the course on that day volunteered to participate in study.

After administering the written questionnaire, semi-structured interview protocols were conducted with 36 prospective teachers in order to get in-depth exploration on prospective elemantary teachers' pedagogical content knowledge especially for knowledge of content and students about possible non- traditional arithmatic operations algorithms held by the elementary students. The semi-structured interview consisted of four parts: (1) Background questions (2) Questions based on nessesities of standard algorithm and non-traditional algorithm for arithmetic operations (3) Questions based on prospective elementary teachers' their own way of strategies to solve arithmatics operation. Trustworthiness is fundemental to judge the quality of the qualitative research. In this study, data triangulation and method triangulation were used to increase the credibility and dependability of the research study. Both questionnaire and interview protocols as different types of data collection tools including were used to analyses consistency among the cases. Furthermore, informant feedback, asking prospective elemantary teachers to comment on the accuracy of questionnaires and transcriptions. In reporting the analysis of the research findings, low inference descriptors, using the phrases that were very close to the participants' wordings is used. In addition to these, thirty six teacher candidate that is more than one individual as a source of data were used. These were the evidences on increasing the credibility of the given case study. Furthermore, investigator triangulation, the use of multiple investigators interpreting the data is used.

RESULTS

Prospective elementary school teachers' interpretation of non-traditional arithmetic operations were analysed systematically. Based on analysis of questionare and semi-structured interview data, prospective elementary school teachers' interpretation of non-traditional arithmetic operations was grouped under two headings. These are:

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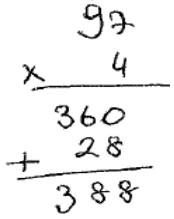
Finding based on prospective elemantary teachers' specialized content knowledge:

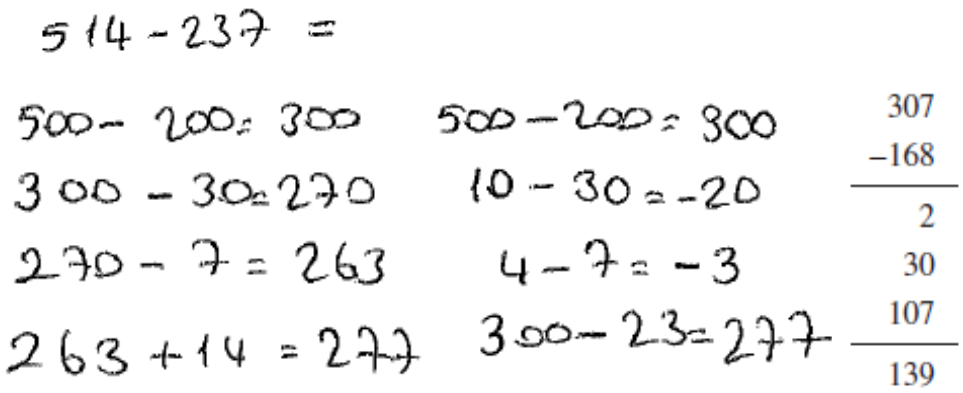
Teacher candidates should able to analyse students errors correctly and determine whether students non-traditional algorithms for arithmatic operation work or not. However, 6 of 36 prospective teachers failed about specialized content knowledge on arithmetics. Some of the prospective teachers interpretaions showed that prospective teachers had **lack of principled mathematical knowledge** on arithmetics operation were given below:



"I do not understand this multiplication. I do not say it is correct. " G-36

"firstly student multiplied 9 with 4 and found 360. This is totally wrong." G-4

As Ball (2007) state, many prospective teachers have learned to carry out this multiplication algorithm without writing down the zeros. This situation shows that prospective teachers are not awere of the reason behind algorithms.



"I think third operation is wrong. The student started subtruction from left (3-1=2)" G-3

"I think students did not understand subtraction conceptually, he probably made subtraction out" M-10)

"These solutions are wrong. The students have to start subtraction from the places of ones, not hundreds" G-6

"This solution is wrong. There is no logic behind it." G-9

These prospective teachers interpretation shows that they are not know counting up strategies and left to right subtraction strategies.

Finding based on prospective elemantary teachers' student- content base pedegojical knowledge

Although 29 of 36 prospective teachers had conceptual knowledge on the operations, they interpreted the students' answers in different ways.

6 of 36 prospective elemantry teachers thought that non-standard algorithms made **operation more complicated.** Some of the prospective elemantry teachers interpretations are given below:

"Although the technique is different than standard algoritms, the answer is correct. The student group the place values of the number than add the place values of the numbers. But I prefer my students to use standard algorithms since this non-traditional method is more confusing., this situation makes finding the correct answer more difficult." G-15

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" I want my student to solve operations with standard algorithms writing the number one under the other, because these non-traditional methods make them confused. " G-13
" I do not say "it is wrong " but I prefer my students to calculate with standard methods which everbody is using, that's why I try to convince them saying that this non-standard methods cause misconceptions and supporting more efford." G-24
" The answer is correct, but I do not give permission to use these non-standard algorithms. Finding the correct answer should not be enough for students, they have to solve the questions as I taught them"G-34

6 of 36 prospective teachers stated that non-standard algorithms **supported easier calculation**. Some of the prospective elemantry teachers interpretations are given below:

"I want my students to suggest non-standard strategies because it makes operation easier and minimizes aritmatic bugs." G-5
"We learned just standart arithmatic operations when we are primary students but I failed finding correct answers because of aritmatics bugs"G-2

7 of 36 prospective teachers thought that non-traditional arithmetic operations **supported conceptual understanding.** Some of the prospective elemantry teachers interpretations are given below:

***"**.....İf student write the number under the other and add them up by prosedurelly, it can cause rote learning. This situation may hinder meaningful learning. " G-11*
" I want my students to think about these non-standard methods and use it, because when they face big numbers they will be awere of breaking these numbers into small numbers and then calculate. In that reason, they can not be afraid of big numbers."G-2
" Using these kind of non-standard methods may support relational understanding. They can use these strategies for the topics they will learn later."G-14
"I want my students try to solve these non-standard strategies because it support attaining number sense and faster mental calculation. This situation shows that they learn playing with numbers. "G-8

6 of 36 prospective teachers stated that these non-traditional strategies were **time confusing**. Some of the prospective elemantry teachers interpretations are given below:

"After conceptual learning occurs, standard methods should use because I think non-standard methods are time confusing."G-34
" This solution is also correct; however writing the number one under the other and using standard algorithm is more correct because it provide with faster calculation." G-36
"This method correct as well; but it cause consuming time."

One of 36 prospective teachers emphasized on **finding correct result; but not strategies.** He stated that:

"Students may try every strategies. Students show their understanding with finding true answers. If the result is correct we should score it. " E-27

Findings of prospective teachers' preferred strategies for calculation of arithmetics operations:

All of the prospective teachers stated that they had learned arithmetics with traditional algorithms. Just 6 of 36 prospective teachers stated that they used non- tradational methods in calculations. One of the prospective teacher's interpretation is given below.

" I learned arithmetic operation with traditional methods. We can name this learning type as rote learning. Hovewer, starting from my university years, I wanted to forget all procedures which I learned rote way, construct the conceptual knowledge. In these process thinking about non-standard methods and trying it was very useful for me. "K-4

Findings of prospective teachers' opinion about drawbacks of teaching arithmetic operations with only standart algorithms

15 of 36 prospective teachers stated that traditional algorithms **prevented students multi-dimentional thinking skills and creative thinking.** Some of the prospective teachers' answers in interview are given below.

"Mathematic education should also support development of flexiable reasoning, in particular towards arithmetics operations. Students should share alternative expressions and strategies in mathematics

education. However, standard algorithms put obstacles into students way of multidimentional thinking." M-1

"if we teach only standard algorithms for operations, we restrain students creativity and we limit their opportunity to play with numbers. G-8

"Not being openminded and not looking for different strategies may hinder thinking in practical ways."G-21

"It may kill students creative thinking and prevent thinking mathematics in flexiable ways."G-31

11 of 36 prospective teachers stated that teaching operations with traditional algorithms could cause **rote learning.**

"It can limit conceptual understanding. The students may not relate operations eachother ".G-9

"I think it is kind of rote learning. İt prevent students analysis the numbers."G-10

"I think, operations should be taught firstly conceptually then should be refered procedurel knowledge to solve the operation faster way. This provide aganist rote learning."G-11

3 of 36 prospective teachers stated that traditional algorthms made **errors increased.**

"I think students have difficulty in learning standard algorithms, that is why they have many mistakes in arithmetics." G-33

3 of 36 prospective teachers stated that traditional algorithms for operations **prevented students from mental arithmetic skills.**

"If we teach students just standard algorithms, they may have difficulty in making calculation in their daily life. They may always need pen and pencil for calculations. "G17

3 prospective teachers said that it **"does not create any problems"**

" It should be tought a standard methods which everybady accept. " M-27

"I am not sure, it may not cause any problems."M-7

" Students calculete the operations faster thankss to standard operations."G-13

DISCUSSION AND CONCLUSION

Students have the capacity to construct mathematical ideas and suggest strategies. Because of high stakes test pressure on students and teachers, teachers tend to teach mathematics not conceptually but procedurelly. However, 2013-2014 elemantary education in mathematics programmes emphasizes on constructing students' their own way of strategies for operations. So; activities which teachers will use, should be convenient with curriculum standards. In that sence teachers and prospective teachers should support students non-traditional arithmetic operations.

One of the reserach findings shows that prospective teacher candidates have lack of spesified content knowledge on arithmetic operations. They have limited conceptual knowledge on arithmetic operations behind procedures. So prospective teachers' interpretation of non traditional arithmetic operation is effected by their spesifid content knowledge. More specially, the prospective teachers who have limited conceptual knowledge on operations stated that just teaching with only standard algorithms is enough for elemantry teachers. To persuade prospective teachers to support students non-traditional algorithms, accurate spesified content knowledge is required. This findings is parellel with Ball (1988) findings.

Although all of the prospective teachers state that they had learned arithmetics with standard algorithms, many of them suggest that non-traditional methods is more convenient for mental computations. And majority of them also stated that supporting students non-traditional algorithms have many advantages such as increasing conceptual understanding, creativity, easier mental calculation, number sense etc. In that sense, prospective teachers have awereness of importance of non-traditional methods, although they had learned arithmetic operations with just standard algorithms.

While taking teaching mathematics 1 course, teacher candidates do not have opportunities to take school observation and teaching experience course. They do not have real observation about student- content knowledge. Some of the prospective teachers stated that standard algorithms were easier for elementary

students so if we teach students non-traditional algorithms, the learning process might be more difficult. They think that students might have difficulty in understanding and producing non-traditional algorithms. I think if they observed elemantry students or had teaching experience they would state that students have capacity to think about and use non-traditional strategies for operations. As a suggestion, school observation course may be required for elementary education program in the first year of their training or teacher educator may provide teacher candidates with observation or cases and students thought process. So prospective teachers will have more opportunities for observing and having interview with students to understand their strategies for operations.

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