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## Dynamic Analysis of Suspension Bridges and Full Scale Testing

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### Abstract

The first part of the study deals with an iteration scheme for the nonlinear static analysis of suspension bridges by means of tangent stiffness matrices. The concept of tangent stiffness matrix is then introduced in the frequency equation governing the free vibration of the system. At any equilibrium stage, the vibrations are assumed to take place tangent to the curve representing the force-deflection characteristics of the structure. The bridge is idealized as a three dimensional lumped mass system and subjected to three orthogonal components of earthquake ground motion producing horizontal, vertical and torsional oscillations. For purposes of illustration the results of analytical and experimental analyses for the Istanbul Bogazici Bridge have been presented.

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*Keywords:* Suspension bridges, dynamic analysis, tangent stiffness, cable structures, bridge testing

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### 1. Introduction

The suspension bridge is a highly nonlinear three dimensional structure. As a consequence, in dynamic studies the governing nonlinear equations of motion are frequently simplified by introducing assumptions which linearize these equations (Konishi and Yamada, 1960). These simplifying assumptions may however be avoided, and the nonlinear behaviour of the structure may thereby be taken into account in both static and dynamic analyses, by using an iterative solution employing tangent stiffness matrices. The iterative scheme has been successfully applied previously by a number of authors in connection with the

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static analysis of suspension bridges (Brotton 1966; Tezcan 1966). In this paper the same operation is extended to solve the dynamic response problem of suspension bridges, which are idealized as three dimensional lumped mass systems vibrating due to earthquake ground motions.

The method proposed for the nonlinear vibration analysis of suspension bridges involves two distinct steps. Under the static action of the dead and live loads the equilibrium configuration and the internal stress resultants of all constituent elements of the structure are first determined through an iteration routine based on the *Newton-Raphson* method. The vibration of any point in the bridge, with respect to the static equilibrium position however, is assumed to take place along the tangent to the curve defining the force-deflection characteristics of that point. Once the fundamental dynamic properties are determined, the response spectrum concept can be used in conjunction with classical modal analysis to evaluate the seismic forces acting on suspension bridges during earthquakes.

## 2. Frequency analysis by tangent stiffnesses

The dynamic analysis of discrete mass structures is a topic which has received extensive treatment in the literature (Housner, 1953; Hurty and Rubinstein, 1964). The nonlinear behaviour of suspension bridges during vibration about any static equilibrium configuration may be accounted for by replacing the linear stiffness matrix of the system,  $[K]_o$ , by a tangent stiffness matrix,  $[K]_T = [K]_o + [K]_g$ . This is equivalent to assuming that at any equilibrium stage the vibration of any point in the bridge takes place along the tangent to the curve representing the force-deflection characteristics of the point. This idea of tangential vibration is illustrated in *Fig. 1*. Accordingly, the frequency determinant becomes

$$\text{Det} | [K]_T - \omega^2 [M] | = 0 \quad (1)$$

where,  $M$  = mass matrix, and  $\omega$  = the natural frequency of the system in any one of its normal modes.  $[K]_T$  depends on the strains and the internal forces developed in the members at the static equilibrium position. The eigenvalues,  $\omega$ , as well as the eigenvectors, can be obtained from a solution of *Eq. 1* using routine computer programs.

## 3. Idealization of the bridge

Depending on the memory capacity of the computer available, the suspension bridge may be idealized as a plane or space frame composed of a series of straight and curved line elements. While the plane frame idealization may be used for the study of the response to vertical and longitudinal ground motions, the three dimensional idealization is desirable for a realistic investigation of the torsional and lateral vibrations of the deck due to ground motion perpendicular to the deck centerline. The main cable and hangers are considered as pure axial force members of constant cross – section, while the deck is assumed to be composed of beam – column elements between hangers. The influence of hanger extensions, cable point loads, degree of fixity at the tower base, stability coefficients due to compressive forces in the bending of tower continuity of the deck across the towers, and variations in moments of inertia can easily be taken into account.

Distributed consistent mass matrices, rather than the lumped masses, have been used for each structural element during the full scale 3D-analyses.

## 4. Modes of vibration

An earthquake may excite a suspension bridge in any one or a combination of the three fundamental types of vibration one about each axes XYZ. At any rate, a full 3-D modeling and analysis of the bridge will already accommodate all modes of vibration and will output the various mode shapes and frequencies. Torsional vibration of the bridge deck, coupled with a lateral vibration of the towers, is due to horizontal ground motion perpendicular to the centerline of the bridge. The torsional vibration is essentially a combination of the vertical and lateral motion of the bridge deck. Such vibrations may also be developed due to lateral wind loading. Vertical vibration of the bridge deck, coupled with a horizontal vibration of the towers, is due to horizontal ground motion parallel to the centerline of the bridge. Vertical vibration of the bridge deck, coupled with a horizontal vibration of the towers (*in longitudinal direction*) is due to vertical ground motion.

In all cases, vertical (*axial*) vibrations of the towers and also longitudinal (*axial*) vibrations of the bridge deck may be neglected, since their effects are relatively small. These three types of vibrations should be definitely taken into account when performing an earthquake analysis of 3D- suspension bridges. Different aspects of this problem have been discussed in the literature for suspension bridges (Jones and Spartz, 1991 ; Harichandran et al. 1991 ; Abdal et al. 1992 and also for cable stayed bridges Fleming and Egesel, 1980 ; Zribi et al. 2006).

## 5. Analyses and full scale testing

The ambient wind vibrations and the forced vibration test results of the *Istanbul Bogazici Bridge* (Figure 2), are the successful examples for the correlation of analytical and experimental studies. A series of experimental studies have been conducted, under the general supervision of Tezcan *et al.* 1974 . Strain gauge readings were taken at a number of locations on the orthotropic deck, towers and hangers, when the carriage ways between the two towers were loaded with heavy trucks i.e. up to three fourths of the bridge capacity. The deflections of the deck, under this particular loading, were also determined by means of precise leveling. The maximum centerline deflection was measured to be *0.76 m*, as verified by the 3D nonlinear analytical calculations described above.

Three different sets of seismometers *a)* by İstanbul Kandilli Observatory team, *b)* by the team of seismologists from the Earthquake Engineering Institute of Skopje and *c)* by *Mr. Alkut Aytun*, a Turkish seismologist, have been installed over the deck and tower in order to record the ambient wind vibrations of the bridge. The inversed fourier transform technique has been used to determine the fundamental periods of vibration. Locations of seismometers and the 2D – mathematical model of the bridge are shown in (Figure 3). The results of the ambient vibration tests are very close to those reported earlier by Brownjohn et al. 1989.

Synchronised twin shakers of the type *GSV-100 Teledyne, USA* supplied by the Skopje Institute were welded at midspan and quarterspan points of the deck, and the forced vibrations were also recorded. The fundamental periods of vibration for a variety of relatively higher modes were determined together with the values of  $\beta$  = critical damping ratio. Most of the results and key parameters obtained from these tests, including those obtained from the wind tunnel tests at the National Physical Laboratories, Teddington, England are listed in *Table1*. Similar analytical studies have been also applied to the newly proposed *Chanakkale Atatürk Bridge* (Tezcan and Arioglu, 1994), and the submerged floating Gibraltar Tunnel (Tezcan and Kaptan, 1995) .

## 6. Conclusions

- For a realistic evaluation of the overall dynamic response of a suspension bridge, a three dimensional idealization is desirable. Such an idealization permits a study of the torsional oscillation of the bridge deck. In fact, significant vibrations of this type were observed due to earthquake ground motion perpendicular to the bridge centerline.
- The general procedures described in this paper may supply useful information in the study of the aerodynamics of suspension bridges.

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## Appendix

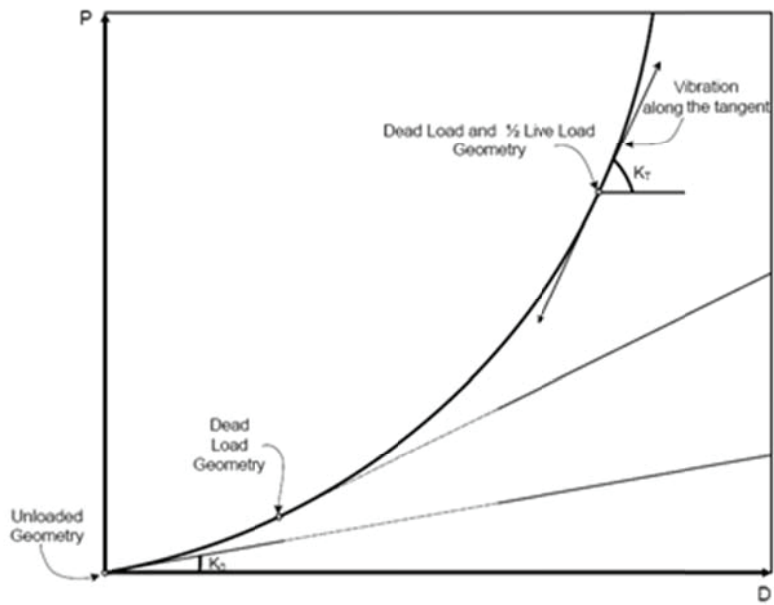


Figure 1 : Vibration along the tangent line



Figure 2 : A general view of Istanbul Bogazici Bridge

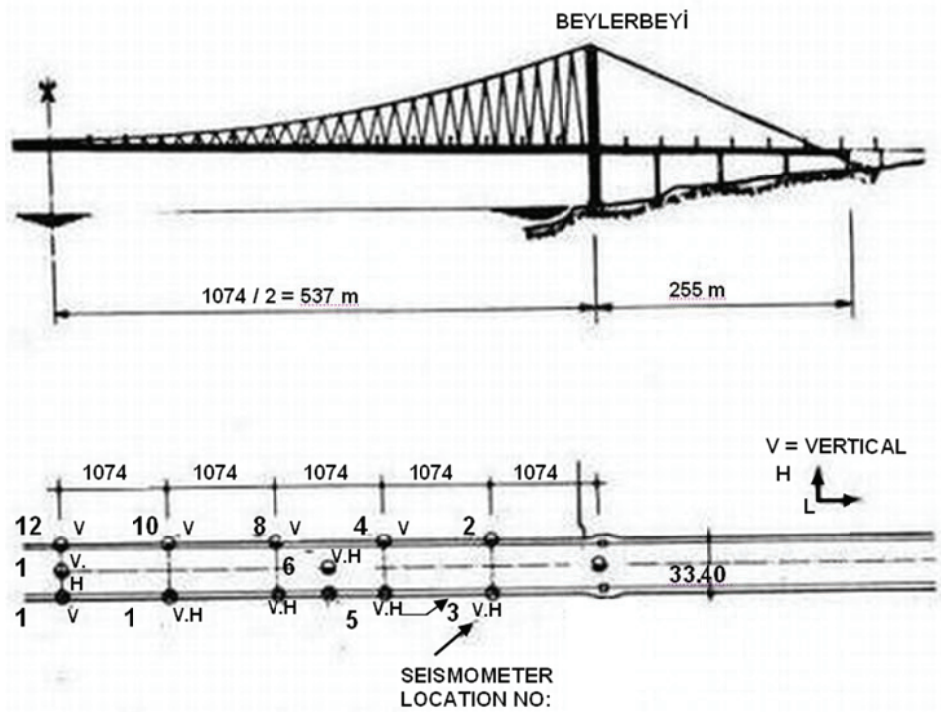


Figure 2 : Location of seismometers on the Bogazici Bridge

Table 1 : Results of Structural Periods (Sec) , Istanbul Bogazici Bridge

TEST METHOD	Longitudinal Vertical	Lateral (Deck)	Lateral (Tower)	Torsional (Deck)
Wind (İstanbul Kandilli Observatory)	6.22	14.15		3.16
Wind (Skopje Institute)	6.24			3.02
Wind (Mr. Alkut Aytun)	6.10		2.77	3.00
Forced Vibrations <sup>(1)</sup> (Skopje Institute)	(1)	(1)	(1)	(1)
	( $\beta = \% 1.2$ ) <sup>(2)</sup>		$\beta = \% 4$	
Wind Tunnel Tests <sup>(2)</sup>	6.41 S			3.16
Computer Analyses	<b>6.83</b>	<b>15.48</b>		<b>3.07</b>

<sup>(1)</sup> Shakers were not large enough to excite the Bridge in the first three modes.

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<sup>(3)</sup>  $\beta =$  critical damping ratio