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RESEARCH ARTICLE

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A simple mathematical programming model for countries' credit ranking problem

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Abstract

This study proposes a new mathematical model that ranks the countries according to their investments. The proposed model compares countries based on specified factors and gives a score for each country. To set out the factors, after investigating on quite a lot of related studies, several macroeconomic factors are gathered, and the most important ones are selected for further investigation. An important advantage of the proposed model is that it can be solved manually by managers, investors, or researchers without a need for any professional optimization solver software. The key factor in this research is developing a mathematical model that ranks any set of countries rather than rating them. At the end of pair comparison, countries are ranked from higher to lower scores from the economical point of view. In addition, the accuracy of the obtained result is validated by comparing them with Moody's rating system (August 2016) using Jaccard similarity index. The results of the proposed are compared with multiple criteria decision-making techniques as well.

KEYWORDS

countries credit ranking, countries credit rating, Jaccard similarity index, non-linear programming

1 | INTRODUCTION

For many investors, it is important to compare their choices of capital investment in different countries. To do such comparisons, the potential countries can be rated or ranked by considering economic and political factors. Therefore, the investors can decide more easily where to invest capitals. The rating and ranking of the countries can be obtained from the economic problems such as credit rating (ranking) problems of the countries. In brief, the credit rating problem takes a set of countries, a set of economic and political factors and performance of each of the countries in terms of factors as inputs of the problem. Then, as an output, the problem defines some rating classes and assigns each country to one of the classes. Because low rating scales are associated with losses for investor, it makes it important for them to measure or estimate country credit rating (Eichler & Hofmann, 2013). On the other hand, the credit ranking problem takes the same inputs and then ranks the countries from the best one to the worst. It is necessary to mention that most of the studies focused on credit rating of the countries instead of ranking them. Therefore, the major part of this work deals with the studies of credit rating of the countries.

In the past, only famous agencies such as Standard & Poor's (S&P's), Moody's, and Fitch were publishing reports about countries' credit assessments that affect the future economic outlook of countries (see Busse & Hefeker, 2007; Butler & Fauver, 2006; Essers, 2013). However, nowadays, credit rating agencies are not the only provider for rating information. In fact, researchers in some fields such as economics, operations researchers, financial management, and statisticians propose valuable approaches to rate countries (Mirzaei & Vizvari, 2011). Moreover, there are evidences indicating that the credit rating agencies often disagree about credit quality, and they rate same countries with one or two notches on the finer scale (Hill, Brooks, & Faff, 2010). Up to now, many mathematical-based approaches are suggested by researchers (see Hirth, 2014; Seitz & La Torre, 2014; Zopounidis & Doumpos, 2000; Zopounnidis & Doumpos, 2002), whereas some scholars proposed using probabilistic and stochastic methods (Aouni, Colapinto, & La Torre, 2014; Gonzalez & Hinojosa, 2010; Hu, Kiesel, & Perraudin, 2002; Kuma & Rao, 2015; Pantelous, 2008). Also, some other researchers (see Afonso, 2003; Cantor & Packer, 1996; Galindo & Tamayo, 2000; Hammer, Kogan, & Lejeune, 2011; Monfort & Mulder, 2000) tried to utilize different methods to assess the financial risk levels and countries credit ranking. Cantor and Packer (1996) applied ordinary least square regressions to a linear representation of the ratings for 45 countries. Afterward, this approach was used similarly by Monfort and Mulder (2000), Afonso (2003), and Butler and Fauver (2006). Although their approach has a good predictive power in estimating the determinants of ratings, it faces some critiques (Afonso, Gomes, & Rother, 2011). To get over the critiques, some other researchers used probabilistic methodology. For instance, Hu et al. (2002) employed order response model for country credit ratings. There are some other studies that used special methodologies such as logical analysis of data (LAD). Hammer et al. (2011) developed a combinatorial nonrecursive model for country risk rating. They utilized combinatorial-logical technique of LAD to develop the model. Both economic-financial and political variables are considered as an input. The results are compared with country risk ratings provided by S&P, and they indicated that the proposed model has 95% accuracy.

In order to rate countries from credit point of view, it is important to identify main influencing factors on country credit rating. Cantor and Packer (1996) mentioned that rating can be done by a few set of factors, which are gross domestic product (GDP) growth, inflation, per capita income, levels of income development, and history income. There is a great existing literature on this subject, but still, there are some disagreements on selected factors. However, there are evidences that prove some factors are common to the majority of studies (see Afonso & Nunes, 2015). Those factors that appear more important in the literature are as follows: GDP per capita or GDP growth rate (Afonso, 2003; Hischer & Nosbusch, 2010), public deficit (Laubach, 2009), monetary policy (Gruber & Kamin, 2010), current account balance (Amira, 2004), and debt budget balance (Baldacci & Kumar, 2010). Although both economic and political factors influence on country credit rating, it is clear from the previous studies that macroeconomic factors have the main effect. For example, Schumacher (2014) determined the short-term relationship between macroeconomic variables and county rating using a panel vector autoregressive approach. He discovered that there is a significant two-way interaction between the macroeconomic factors and changes in sovereign's rating. Karolyi (2015) believed that there are six indicators that affect building the emerging markets' risk. Market capacity constraints, operational inefficiencies, foreign accessibility restrictions, corporate opacity, limits to legal protections, and political instability are those six indicators.

In a nutshell, up to now, many different models are proposed for country credit/risk ratings. However, there is no single model that is able to rate countries in the same way as famous agencies such as S&P, Moody's, and Fitch do. All of the mentioned methodologies rate the countries, not rank them. That is why, in this study, a new mathematical model to rank the countries is developed. The proposed model compares countries based on specified factors and gives a score to each country. An advantage of the proposed model is that it can be solved manually by managers and investors without a need for any professional optimization solver software. To the best of our knowledge, this study is the first to rank the countries (in descending order) from their credit point of view. To do so, after investigating on quite a lot of related studies, several macroeconomic factors are gathered, and the most important ones are selected for further investigation. The model allows the researchers to obtain country credit ranking, a point of distinction from other studies, which have generally rated countries and categorizes them into prespecified rating scales. In addition, the model accuracy is validated by comparing the obtained empirical results with Moody's rating system.

The sections included in this paper are as follows. The problem definition and data collection are discussed in the Section 2. In Section 3, a non-linear mathematical model is proposed for credential ranking of countries. Section 4 presents the computational experiments. Section 5 is devoted to obtain results. Finally, conclusion and discussion are presented in Section 6.

2 | PROBLEM DEFINITION AND DATA SET

2.1 | Problem definition

The same as the rating reports that are published by many agencies such as S&P, Moody's, and Fitch, countries can be rated according to their investing opportunities. It is important to note that these agencies rate countries by considering many factors such as economic, political, agricultural, and infrastructure. There are many mathematicians, economists, analysts, and experts whose work corporates with the agency (Moody's), so that different methodologies are utilized for this reason. The ratings obtained from the agencies give a better rate to the countries that provide better economic environment in addition to other factors for investors to obtain more profit. However, there are some countries that have the same rating scale, which means the economic outlook of those countries are similar to each other, but not identical. In this case, it is difficult to compare these countries with each other because they receive the same rating scale. In such situation, a new approach is needed to find the better alternative (country) among those of the same rate. As a solution to such difficulty, countries can be ranked from the best one to the worst one from credit point of view. The better economic environment can be exampled when a country has better performance in the economic factors that are effective in economic growth of the countries. In this case, each economic factor has a weight among all the factors. Therefore, the countries having better performance in the most important economic factors may have higher chance to obtain a better position in the credit ranking of the countries. In general, the performances of a country considering the weights of the factors can be used to calculate a score for the performance of that country.

The obtained scores for the countries can be used to determine the credit ranking of the countries. As a result, in this study, a ranking method is proposed to compare countries and rank them in descending order to select the best county or the best order of counties for future investment.

2.2 | Data set of the problem

In this study, 54 countries are considered for credit ranking problem. The countries are to be ranked based on their performances obtained from 25 economic factors. All the data are obtained from International Monetary Fund (IMF) database. The 54 countries selected have available performance data set in IMF economic database. Tables 1 and 2 show the economic factors and the countries, respectively. In Table 1, the factors indicated by "positive" sign in the parentheses are the factors with favourable performance. On the other hand, for the "negative" factors, lower performance is desired.

TABLE 1 Economic factors selected from IMF database

Factor number	Factor name and type
1	Gross domestic product, constant prices (positive)
2	Gross domestic product, current prices (positive)
3	Gross domestic product based on PPP valuation of country GDP (positive)
4	Gross domestic product based on PPP per capita GDP (positive)
5	Gross domestic product per capita, current prices (positive)
6	Gross domestic product based on PPP share of world total (positive)
7	Total investment (positive)
8	Gross national savings (positive)
9	Volume of exports of goods and services (positive)
10	General government revenue (positive)
11	General government total expenditure (positive)
12	General government net lending/borrowing
13	Current account balance ($; positive)
14	Current account balance (percentage of GDP; positive)
15	Volume of exports of goods (positive)
16	General government primary net lending/borrowing (positive)
17	Inflation, average consumer prices (index; negative)
18	Inflation, average consumer prices (percentage change; negative)
19	Inflation, end of period consumer prices (index; negative)
20	Inflation, end of period consumer prices (percentage change; negative)
21	Volume of imports of goods and services (negative)
22	Volume of imports of goods (negative)
23	Unemployment rate (negative)
24	General government gross debt (negative)
25	Gross domestic product, deflator (negative)

Note. IMF: International Monetary Fund; PPP: purchasing power parity; GDP: gross domestic product.

TABLE 2 The selected countries for the credit ranking problem by their alphabetical order

No.	Country	No.	Country	No.	Country	No.	Country
1	Albania	15	Estonia	28	Kazakhstan	42	Portugal
2	Australia	16	France	29	Korea	43	Romania
3	Austria	17	Germany	30	Kuwait	44	Russia
4	The Bahamas	18	Honduras	31	Latvia	45	Singapore
5	Belgium	19	Hong Kong SAR	32	Malaysia	46	Slovak Republic
6	Belize	20	Hungary	33	Morocco	47	South Africa
7	Bulgaria	21	Iceland	34	Netherlands	48	Sweden
8	Canada	22	Ireland	35	New Zealand	49	Thailand
9	Chile	23	Israel	36	Norway	50	Tunisia
10	China	24	Italy	37	Pakistan	51	Turkey
11	Colombia	25	Jamaica	38	Panama	52	United Kingdom
12	Costa Rica	26	Japan	39	Peru	53	United States
13	Denmark	27	Jordan	40	Philippines	54	Uruguay
14	Egypt			41	Poland		

3 | AN APPROACH TO COUNTRIES CREDIT RANKING

In this section, a non-linear mathematical model is proposed to find optimal weight of each factor, and then a technique is used to rank the countries. The approach is summarized in the following subsections.

3.1 | The mathematical model to optimize weights of the factors

The following notations are used in the model:

- m Number of countries (parameter)
- n Number of economic factors (parameter)
- i Index used for countries (index)
- j Index used for factors (index)
- w_j Importance of factor j (variable)
- x_{ij} Performance of country i in factor j (parameter)
- y_{ij} Normalized performance of country i in factor j (parameter)
- s_i Score of country i (variable)
- P Set of positive economic factors (parameter)
- N Set of negative economic factors (parameter)

The aim of the model is to find the weights of the factors to maximize the total performance of each country. An assumption on the weights of the factors is that w_1, w_2, ..., $w_n \geq 0$. The model is as follows:

$$\max \sum_{j=1}^{n} w_j \left(\sum_{i=1}^{m} y_{ij} \right), \tag{1}$$

subject to

$$\sum_{j=1}^{n} w_j^2 = 1, \tag{2}$$

$$w_j \geq 0 \quad j = 1, \ 2, \ ..., \ n. \tag{3}$$

The objective function (1) maximizes the sum of all normalized performances of the countries weighted of the factors. Constraint (2) forces the Euclidean norm ($\|.\|_2$) of the weights of the factors to be equal to 1, which normalizes the weights. Constraint set (3) guarantees a positive weight for each factor.

The idea of this model is partially taken from simple additive weighting (SAW) method with one exception that in the SAW method, the weights are exogenous and determined by decision-maker, but in this model, those are endogenous and are obtained from the data of the problem. Therefore, using constraint (2), the model is a decision-maker independent model. Meaning that the model is an endogenous-type model and the weight of each factor is determined by the data of the model (normalized performances) endogenously, instead of being determined by a decision-maker. This form of constraint helps us to have easier solution calculations that will be explained later in this section. Instead of this constraint, the constraints $\sum_{j=1}^{n} w_j = 1$ and $\sum_{j=1}^{n} w_j^2 = K$ where K is an arbitrary positive value can be used, where in latter case, $\sum_{j=1}^{n} w_j^{'} = 1$ is obtained where $w_j^{'} = \frac{w_j^2}{K}$.

As the economic factors are classified into two types of positive and negative factors (see Table 1), the normalization of x_{ij} (y_{ij}) is obtained by

$$\begin{cases} y_{ij} = \dfrac{x_{ij} - \min\limits_{i=1,\dots,m}\{x_{ij}\}}{\max\limits_{i=1,\dots,m}\{x_{ij}\} - \min\limits_{i=1,\dots,m}\{x_{ij}\}} & \forall\, j \in P \\ y_{ij} = \dfrac{\max\limits_{i=1,\dots,m}\{x_{ij}\} - x_{ij}}{\max\limits_{i=1,\dots,m}\{x_{ij}\} - \min\limits_{i=1,\dots,m}\{x_{ij}\}} & \forall\, j \in N \end{cases} \tag{4}$$

This normalization is done as the considered factors are of different scales. For example, some of them take values with unit of currency, and some others take ratio values. Therefore, these values are normalized to have no scale. In this way, any comparison and mathematical operation can be done on the data.

Although the model in Section 3.1 is solvable by any optimization solver, use of Euclidean norm in constraint (2) is an advantage for the model to be solved manually without any optimization solver. A solution method to the model (1)–(3) can be proposed applying Lagrange multiplier method. The model helps financial managers and decision-makers to solve the model analytically. Therefore, an analytical solution of the model is obtained by Lagrange multiplier method as described here.

The auxiliary function of the model is constructed as follows:

$$\Lambda(w_1, w_2, \dots, w_n, \gamma) = \left(\sum_{j=1}^{n} w_j \left(\sum_{i=1}^{m} y_{ij}\right)\right) + \gamma\left(\left(\sum_{j=1}^{n} w_j^2\right) - 1\right). \tag{5}$$

Then the equation

$$\nabla_{w_1, w_2, \dots, w_n, \gamma} \Lambda(w_1, w_2, \dots, w_n, \gamma) = 0 \tag{6}$$

must be held, which implies the following derivations:

$$\frac{\partial \Lambda}{\partial w_j} = 0 \Rightarrow w_j = \frac{-\sum\limits_{i=1}^{m} y_{ij}}{2\lambda} \quad j = 1, \ 2, \ \dots, \ n, \tag{7}$$

$$\frac{\partial \Lambda}{\partial \gamma} = 0 \Rightarrow \left(\sum_{j=1}^{n} w_j^2\right) - 1 = 0. \tag{8}$$

Supplying the value obtained for w_j from Equation (7) in Equation (8) results in the following values for λ:

$$\lambda = \begin{cases} + \dfrac{\sqrt{\sum\limits_{j=1}^{n}\left(\sum\limits_{i=1}^{m} y_{ij}\right)^2}}{2} \\ - \dfrac{\sqrt{\sum\limits_{j=1}^{n}\left(\sum\limits_{i=1}^{m} y_{ij}\right)^2}}{2} \end{cases} \tag{9}$$

To determine the value of w_j^* from λ, Equation (10) is used.

$$w_j^* = \frac{\sum\limits_{i=1}^{m} y_{ij}}{\sqrt{\sum\limits_{j=1}^{n}\left(\sum\limits_{i=1}^{m} y_{ij}\right)^2}} \quad j = 1, \ 2, \ \dots, \ n. \tag{10}$$

Remark. As the normalized performance values y_{ij} in the objective function (1) are always non-negative ($0 \le y_{ij} \le 1$), in this maximization-type objective function, the values for w_j^* cannot be negative (these variables are also restricted by constraint set (3)). On the other hand, according to Equation (9), λ can take both positive and negative values. In order to find non-negative value for w_j^*, the negative value of λ is replaced in Equation (7), which gives an optimal analytical solution to the model (1)–(3) by Equation (10). Of course, for the cases that the normalized performance values y_{ij} in the objective function (1) take negative values ($y_{ij} < 0$), the proposed method cannot be used. In such cases, data envelopment analysis (DEA) approaches with negative data (negative DEA) can be useful.

3.2 | Credit ranking procedure for the countries

The optimal weights of the factors obtained from the previous section are used to calculate the score of each country. The score of each country is calculated by the following relation:

$$s_i = \sum_{j=1}^{n} w_j^* y_{ij} \quad i = 1, \ 2, \ \dots, \ m. \tag{11}$$

Finally, the countries are ranked based on the decreasing order of their scores. The procedure of the proposed countries ranking method is summarized in Figure 1.

4 | COMPUTATIONAL EXPERIMENTS

To perform the proposed countries credit ranking approach on the data of Section 2, the performance of each country in each economic factor (x_{ij}) has to be

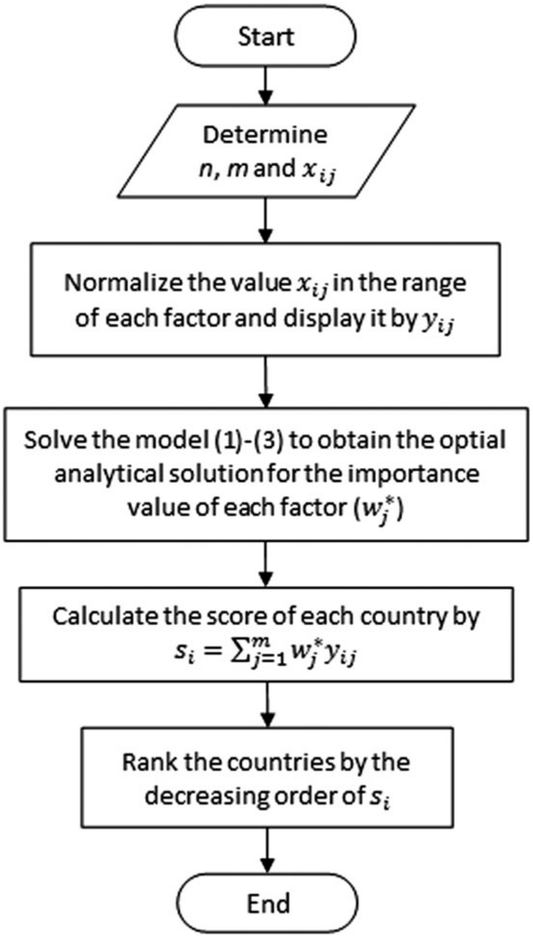

FIGURE 1 Flowchart of the proposed countries credit ranking approach

normalized. The x_{ij} values of all countries in all 25 selected factors are normalized (y_{ij}) by Equation (4). The obtained normalized values of a negative factor, such as unemployment rate of Table 1, for all countries are depicted in Table 3. The same is applied to normalize the performances of the countries in all economic factors of Table 1.

After normalizing the performances of each country in all factors, the model (1)–(3) is solved to obtain the optimal analytical solution of the weight of each factor. Then countries are sorted by decreasing order of their scores obtained by Equation (3). The rankings obtained for the countries are shown by Table 4. In addition to the proposed approach of this study, two well-known approaches of the literature were applied to rank the countries. For this aim, the approaches, for example, the technique for order of preference by similarity to ideal solution (TOPSIS; see Niroomand, Bazyar, Alborzi, and Mahmoodirad. 2018, for more details) and DEA (see Kao, 2010, for more details) were used and their obtained ranking is depicted by Table 4. The differences of the obtained rankings among the proposed approach and the TOPSIS and DEA approaches can be realized from Table 4. It is notable to mention that in the TOPSIS approach, all of the criteria were weighted equally.

5 | ANALYSIS OF THE OBTAINED RESULTS

As mentioned before, the model ranks the countries, it does not rate them. As all rating agencies rate the countries, there is no ranking source to compare the obtained result of this study. In this section, a technique is used to compare the ranking obtained in this study with Moody's rating results.

5.1 | Moody's rating

Moody's agency rates the countries in 21 categories. The categories from the best to the worst ones are titled as Aaa, Aa1, Aa2, Aa3, A1, A2, A3, Baa1, Baa2, Baa3, Ba1, Ba2, Ba3, B1, B2, B3, Caa1, Caa2, Caa3, Ca, and C. These categories can be mapped to numbers 1 to 21, respectively (e.g., class Aaa can be shown by number 1). Therefore, the ranking obtained in Table 4 and their associated Moody's rating is mentioned in Table 5.

5.2 | Comparing the obtained ranks with the Moody's rating

In this section, Jaccard similarity index (Levandowsky & Winter, 1971; Qian, Wu, & Xu, 2011) is used to compare the obtained ranks and the Moody's rating of the countries. Usually, Jaccard similarity index is used to measure the similarity of two given vectors of numbers.

If $X = (x_1, x_2, ..., x_n)$ and $Y = (y_1, y_2, ..., y_n)$ are two vectors such that $\forall i \in \{1, \; 2, ..., n\} : x_i, y_i \geq 0$, then the Jaccard similarity index of these two vectors is shown by $J(X, Y)$ and is calculated by

$$J(X,Y) = \frac{\sum_{i=1}^{n} \min\{x_i, y_i\}}{\sum_{i=1}^{n} \max\{x_i, y_i\}}, \tag{12}$$

where $0 < J(X, Y) \leq 1$..

Obviously, more closer x_i and y_i will result in higher Jaccard similarity index. Therefore, higher values of Jaccard similarity index show more similarity of the vectors.

To calculate Jaccard similarity index of the obtained ranks of this study and the Moody's rating result, the following steps are used:

TABLE 3 Original and normalized values of the performances of the countries in a negative factor for instance

Country	Factor 23 (Negative factor) Unemployment rate		Country	Factor 23 (Negative factor) Unemployment rate	
	x_{ij}	y_{ij}		x_{ij}	y_{ij}
Albania	13	0.488395308	Kazakhstan	5.2	0.812827552
Australia	6.175	0.772273521	Korea	3.125	0.899134847
Austria	5	0.821146327	Kuwait	2.072	0.942933200
The Bahamas	15.513	0.383869894	Latvia	10.679	0.584934698
Belgium	9.1	0.650611430	Malaysia	3	0.904334082
Belize	15.645	0.378379503	Morocco	9.135	0.649155644
Bulgaria	12.5	0.509192247	Netherlands	7.302	0.725397222
Canada	7.027	0.736835538	New Zealand	5.217	0.812120456
Chile	6.05	0.777472756	Norway	3.5	0.883537143
China	4.1	0.858580817	Pakistan	6.882	0.742866650
Colombia	9.3	0.642292655	Panama	4.5	0.841943266
Costa Rica	8.5	0.675567756	Peru	6	0.779552450
Denmark	6.8	0.746277348	Philippines	6.9	0.742117960
Egypt	12.977	0.489351967	Poland	10.213	0.604317444
Estonia	8.459	0.677273105	Portugal	15.65	0.378171533
France	11.044	0.569752932	Romania	7.168	0.730970801
Germany	5.211	0.812370019	Russia	6.2	0.771233674
Honduras	4.5	0.841943266	Singapore	2	0.945927959
Hong Kong SAR	3.052	0.902171200	Slovak Republic	13.856	0.452790949
Hungary	9.425	0.637093420	South Africa	24.742	0
Iceland	3.705	0.875010398	Sweden	8.042	0.694617752
Ireland	11.217	0.562557192	Thailand	0.7	1
Israel	6.7	0.750436736	Tunisia	16	0.363613676
Italy	12.4	0.513351635	Turkey	10.164	0.606355544
Jamaica	14.9	0.409366941	United Kingdom	6.931	0.740828550
Japan	3.944	0.865069462	United States	6.384	0.763580401
Jordan	12.2	0.521670410	Uruguay	6.8	0.746277348

TABLE 4 Complete credit ranking of the countries obtained by the model (2)–(4) and the approaches of literature

Country	The proposed approach Score (s_i)	The proposed approach Ranking	Rank by DEA	Rank by TOPSIS	Country	The proposed approach Score (s_i)	The proposed approach Ranking	Rank by DEA	Rank by TOPSIS
Kuwait	3.315773654	1	9	1	Slovak Republic	2.814209821	28	17	34
Norway	3.277523306	2	1	4	Chile	2.801783584	29	29	31
United States	3.241259953	3	11	6	Colombia	2.801419059	30	32	30
Australia	3.195057229	4	30	7	Peru	2.799650872	31	34	27
Sweden	3.169999345	5	3	10	Iceland	2.796314688	32	50	24
Germany	3.131483600	6	13	3	Poland	2.795204234	33	35	37
Netherlands	3.071136342	7	5	9	Romania	2.785320918	34	37	36
Denmark	3.067495471	8	4	14	Portugal	2.777365135	35	33	35
Austria	3.041618259	9	8	16	Russia	2.756823570	36	38	51
Singapore	3.023972574	10	2	5	Panama	2.712134283	37	31	42
Canada	2.994602219	11	21	13	Japan	2.706568770	38	40	15
United Kingdom	2.991626210	12	16	38	Morocco	2.652666926	39	41	41
Belgium	2.990879895	13	10	22	Philippines	2.612469450	40	43	26
Korea	2.974668703	14	6	8	The Bahamas	2.610235025	41	44	43
Israel	2.962254738	15	20	19	Uruguay	2.608909622	42	36	39
Malaysia	2.951425741	16	14	18	Belize	2.602747422	43	39	40
France	2.950286251	17	26	25	Jordan	2.594026838	44	47	44
China	2.938135976	18	19	2	Albania	2.577210259	45	46	46
Italy	2.920070136	19	7	12	Tunisia	2.503229466	46	49	47
Ireland	2.893641324	20	12	20	Kazakhstan	2.461969533	47	42	21
Bulgaria	2.879062995	21	22	32	Honduras	2.319429412	48	25	53
Latvia	2.876415201	22	18	28	Turkey	2.313966745	49	48	49
Hong Kong SAR	2.859039918	23	24	11	South Africa	2.307722006	50	51	48
New Zealand	2.856059005	24	23	17	Costa Rica	2.295971450	51	45	52
Hungary	2.847540010	25	28	29	Pakistan	2.238206578	52	54	45
Thailand	2.841395984	26	27	23	Jamaica	1.902585023	53	53	50
Estonia	2.816685299	27	15	33	Egypt	1.844794771	54	52	54

TABLE 5 Obtained rank versus Moody's rates of the countries

Country	Rank	Moody's rating	Country	Rank	Moody's rating	Country	Rank	Moody's rating
Kuwait	1	Aa2 (3)	Italy	19	Baa2 (9)	Panama	37	Baa2 (9)
Norway	2	Aaa (1)	Ireland	20	Baa1 (8)	Japan	38	Aa3 (4)
United States	3	Aaa (1)	Bulgaria	21	Baa2 (9)	Morocco	39	Ba1 (11)
Australia	4	Aaa (1)	Latvia	22	Baa1 (8)	Philippines	40	Baa3 (10)
Sweden	5	Aaa (1)	Hong Kong SAR	23	Aa1 (2)	The Bahamas	41	Baa1 (8)
Germany	6	Aaa (1)	New Zealand	24	Aaa (1)	Uruguay	42	Baa2 (9)
Netherlands	7	Aaa (1)	Hungary	25	Ba1 (11)	Belize	43	Caa2 (18)
Denmark	8	Aaa (1)	Thailand	26	Baa1 (8)	Jordan	44	B1 (14)
Austria	9	Aaa (1)	Estonia	27	A1 (5)	Albania	45	B1 (14)
Singapore	10	Aaa (1)	Slovak Republic	28	A2 (6)	Tunisia	46	Ba3 (13)
Canada	11	Aaa (1)	Chile	29	Aa3 (4)	Kazakhstan	47	Baa2 (9)
United Kingdom	12	Aa1 (2)	Colombia	30	Baa2 (9)	Honduras	48	B3 (16)
Belgium	13	Aa3 (4)	Peru	31	A3 (7)	Turkey	49	Baa3 (10)
Korea	14	Aa3 (4)	Iceland	32	Baa3 (10)	South Africa	50	Baa1 (8)
Israel	15	A1 (5)	Poland	33	A2 (6)	Costa Rica	51	Baa3 (10)
Malaysia	16	A3 (7)	Romania	34	Baa3 (10)	Pakistan	52	Caa1 (17)
France	17	Aa1 (2)	Portugal	35	Ba1 (11)	Jamaica	53	Caa3 (19)
China	18	Aa3 (4)	Russia	36	Baa1 (8)	Egypt	54	Caa1 (17)

TABLE 6 Vectors A, B, and C for the ranking obtained by the model (2)–(4)

Vector A	Vector B	Vector C	Vector A	Vector B	Vector C
Kuwait	3	1	Slovak Republic	6	8
Norway	1	1	Chile	4	8
United States	1	1	Colombia	9	8
Australia	1	1	Peru	7	8
Sweden	1	1	Iceland	10	8
Germany	1	1	Poland	6	9
Netherlands	1	1	Romania	10	9
Denmark	1	1	Portugal	11	9
Austria	1	1	Russia	8	9
Singapore	1	1	Panama	9	9
Canada	1	1	Japan	4	9
United Kingdom	2	2	Morocco	11	10
Belgium	4	2	Philippines	10	10
Korea	4	2	The Bahamas	8	10
Israel	5	3	Uruguay	9	10
Malaysia	7	4	Belize	18	10
France	2	4	Jordan	14	11
China	4	4	Albania	14	11
Italy	9	4	Tunisia	13	11
Ireland	8	4	Kazakhstan	9	13
Bulgaria	9	5	Honduras	16	14
Latvia	8	5	Turkey	10	14
Hong Kong SAR	2	6	South Africa	8	16
New Zealand	1	6	Costa Rica	10	17
Hungary	11	7	Pakistan	17	17
Thailand	8	7	Jamaica	19	18
Estonia	5	8	Egypt	17	19

Step 1: Three vectors containing 54 element each are considered:

Vector A contains the countries based on the order of the ranking obtained by model (2)–(4) as shown by Table 3.

Vector B contains the rating of the countries by Moody's for the order of countries of Vector A.

Vector C contains increasing order of the elements of Vector B. Vector C is actually the expected rating vector of the ranked countries. For example, in a perfect ranking, it is expected that all 11 countries having rate 1 be ranked as first 11 countries and so on.

Step 2: Equation (2) is used to calculate Jaccard similarity index of vectors B and C, to obtain the similarity index of the obtained rank by model (2)–(4) and Moody's rating.

Vectors A, B, and C of 54 countries of this study, based on the obtained ranks from the proposed model (2)–(4), are represented in Table 6.

To measure Jaccard similarity index of the obtained ranks and the Moody's rating of the countries, vectors B and C of Table 6 are applied. The index is calculated as follows:

$$J\left(\text{Ranking-Rating}, \text{Moody}'\text{s Rating}\right) = \frac{\min\{3,\ 1\} + \min\{1,\ 1\} + \ldots + \min\{19,\ 18\} + \min\{17,\ 19\}}{\max\{3,\ 1\} + \max\{1,\ 1\} + \ldots + \max\{19,\ 18\} + \max\{17,\ 19\}} = 0.7404.$$

Applying similar procedure shows that Jaccard similarity index of the obtained ranking by TOPSIS and Moody's rating is 0.6949, and Jaccard similarity index of the obtained ranking by DEA and Moody's rating is 0.6913. It means that when comparing with Moody's rating, the proposed approach of this study outperforms the TOPSIS and DEA approaches.

6 | CONCLUDING REMARKS

This study proposes a new approach to rank countries by considering different macroeconomic factors. Twenty-five economic factors are selected as parameters for comparing 54 counties. Consequently, a non-linear mathematical model proposed to find optimal weight of each factor, and based on the obtained results, the countries are ranked in descending order. To validate and verify the model, the obtained ranking is compared with Moody's rating system (August 2014) by Jaccard similarity index. It can be concluded that the rating system has shown more than 74% similarity with the rating system of Moody's. Although the proposed ranking model is different from the rating methods of agencies, but by considering both results, it can be concluded that the developed ranking method is accurate enough to be used instead of the rating system to assess and compare countries. This claim is justified by the results of Jaccard similarity index test. The misclassification (difference between raking result and Moody's rating) is unavoidable because of methodology and factors selection. The ranking in this study has similarity to Moodys' rating, and just few dissimilarities occurred. One of the main reasons for dissimilarity is that political factors are not considered in this study. The other reason is the size of the set for ranking the countries that are used. This can be explained in this way that the differences are reasonable by considering limitation in factors selection and simplicity of the model that is used for ranking. Also, the proposed model is useful for ranking the countries that have received identical rate scales from rating agencies.

Furthermore, another purpose of this study is to introduce a novel model to assess and compare countries with each other with minimum number of factors that are available. In addition, the non-linear model that is developed in this study can be solved manually or by MS Excel easily, and there is no need for complicated solver software to run and solve the model.

As future study on this topic, the proposed model can be used in different ways. Individually, when there is an uncertainty in considering a set of alternatives (countries) for investment, with the help of the model, it is possible to rank the alternatives and find the best one/s for investment consideration.

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