Magnetic dipole moments of the $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ states in light-cone QCD

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The magnetic dipole moments of the $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ states are extracted in the framework of the light-cone QCD sum rules. In the calculations, we use the hadronic molecular form of interpolating currents and photon distribution amplitudes to get the magnetic dipole moments of the $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ tetraquark states. The magnetic dipole moments are obtained as $\mu_{Z_c} = 0.66^{+0.27}_{-0.25}$, $\mu_{Z_c^1} = 1.03^{+0.32}_{-0.29}$, $\mu_{Z_{cs}} = 0.73^{+0.28}_{-0.26}$ and $\mu_{Z_{cs}^1} = 0.77^{+0.27}_{-0.25}$ for the $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ states, respectively. We observe that the results obtained for the $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ states are large enough to be measured experimentally. As a by product, we predict the magnetic dipole moments of the neutral $Z_{cs}(4000)$ and $Z_{cs}(4220)$ states. The results presented here can serve to be helpful knowledge in experimental as well as theoretical studies of the properties of hidden-charm tetraquark states with and without strangeness.

I. INTRODUCTION

The standard hadrons, made of the strongly interacting quarks and gluons, are mesons and baryons. There have been proposed particles having configurations out of the standard hadrons, and some of them have been observed in experiments. These are exotic particles including XYZ particles, hybrids, meson molecules, tetraquarks, pentaquarks etc. X(3872) is the first of four quark structures which contains a charm quark, and it was experimentally observed in 2003 by Belle [1]. After X(3872), charged or uncharged, many exotic hadrons were observed. To date, there are six members in the group of the charged charmed exotic resonances, $Z_c(3900)$, $Z_c(4020)$, $Z_c(4050)$, $Z_c(4200)$, $Z_2(4250)$ and $Z_c(4430)$, reported in decays into final states containing a pair of light and charm quarks [2–11]. They cannot be classified as traditional charmonium states because of their electric charge, and they must be exotic states with a minimum quark content $c\bar{c}u\bar{d}/c\bar{c}d\bar{u}$. Many attempts have been made to figure out the substructure of these exotic states, but the nature of most of

them is still unclear. As tetraquark states with quark contents $c\bar{c}u\bar{d}/c\bar{c}d\bar{u}$, a family of these charged exotic states was generally studied as diquark-antidiquark and molecular pictures [12–25].

Very recently, three charged hidden-charm tetraquark states with strangeness were reported by the BESIII [26] and LHCb [27] Collaborations. The spin-parity of these states are assumed to favor $J^P = 1^+$, and their quark composition is most likely $c\bar{c}s\bar{u}/c\bar{c}\bar{s}u$. Their masses and widths are

$$Z_{cs}(3985)\colon \mathrm{M} = 3982.5 \pm 2.1^{+1.8}_{-2.6} \text{ MeV},$$
$$\Gamma = 12.8^{+5.3}_{-4.4} \pm 3.0 \text{ MeV}, \tag{1}$$

$$Z_{cs}(4000)\colon \mathrm{M} = 4003.0 \pm 6.0^{+4.0}_{-14.0} \text{ MeV},$$
$$\Gamma = 131.0 \pm 15 \pm 26 \text{ MeV}, \tag{2}$$

$$Z_{cs}(4220)\colon \mathrm{M} = 4216.0 \pm 24.0^{+43}_{-30} \text{ MeV},$$
$$\Gamma = 233.0 \pm 52^{+97}_{-73} \text{ MeV}. \tag{3}$$

The $Z_{cs}(3985)$ and $Z_{cs}(4000)$ states have very different decay widths and can be considered as two different states, in spite of being very close in mass. This important difference means that $Z_{cs}(3985)$ and $Z_{cs}(4000)$ can have different origins. However, there is a possibility that both states are the same, but their masses and decay widths may not be consistent because of experimental resolution and/or coupled-channels effects. Moreover, the $Z_{cs}(3985)$ and

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$Z_{cs}(4220)$ particles can be interpreted as strange partners of the $Z_c(3900)$ and $Z_c(4020)$ states, respectively. Clarification of the aforementioned possibilities regarding these states is among the questions that need to be answered. These discoveries triggered interesting theoretical examinations in the context of various approaches aimed to account for substructures of the $Z_{cs}(3985)$, $Z_{cs}(4000)$ and $Z_{cs}(4220)$ states and calculate their parameters [28–51]. Most of these studies are related to the calculation of spectroscopic parameters.

The electromagnetic properties like masses and decays are the important parameters of a hadron, which are measurable and computable. Therefore, the examination of these properties is important to understand their substructure, nonperturbative picture and dynamics of the exotic states. In the present study, we calculate the magnetic dipole moments (MDMs) of the $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ (hereafter, we show these states as Z_c, Z_c^1, Z_{cs} and Z_{cs}^1, respectively) tetraquark states by considering them as the molecular picture in the framework of light-cone QCD sum rules (LCSR). The LCSR is a powerful method in investigating the exotic hadron characteristics and has been performed successfully to compute the masses, decay constants, magnetic moments, etc. In the LCSR, the operator product expansion (OPE) is carried out over twist near the light cone. In this case, there come into view matrix elements of nonlocal operators between the vacuum and photon state. These matrix elements are expressed with respect to the distribution amplitudes (DAs) of the photon [52–54]. There are a few studies in the literature where the MDMs of the hidden-charm tetraquark states were extracted [55–59].

The outline of our paper is as follows. In Sec. II, we acquire the QCD sum rules for the MDMs of the tetraquark diquark-antidiquark and molecular states. Section III is devoted to the numerical analysis and discussion on the obtained results. The obtained results are summarized and discussed in Sec. IV. The explicit expressions of the magnetic moment of the $Z_{c(s)}$ tetraquark states are given in the Appendix.

II. FORMALISM

It is well known that in the LCSR approach, the correlation function is obtained in two different ways: hadronic and QCD sides. In order to obtain the expression from the hadronic side, the correlation function is saturated with the same quantum numbers as the interpolating currents. In the QCD side, the correlation function is obtained in deep Euclidean region, $q^2 \ll 0$, with the help of the OPE. As usual, in the LCSR approach, in order to suppress the contribution of the continuum and higher states, Borel transformation and continuum subtractions are applied. The sum rules for the MDMs of the $Z_{c(s)}$ states are obtained equating both representations of the correlation function via the quark-hadron duality ansatz.

We started with a two-point correlation function to obtain the sum rules for the MDMs of $Z_{c(s)}$, which can be written as

$$\Pi_{\mu\nu}(p,q) = i \int d^4x e^{ip\cdot x}\langle 0|\mathcal{T}\{J_\mu(x)J_\nu^\dagger(0)\}|0\rangle_\gamma, \quad (4)$$

where $J_{\mu(\nu)}(x)$ is the interpolating current of the $Z_{c(s)}$ states, and γ is the external electromagnetic field. In the molecular picture with quantum numbers $J^P = 1^+$, it is given as

$$J_\mu^{Z_c}(x) = \frac{1}{\sqrt{2}}\{[\bar{d}_a(x)\gamma^\alpha c_a(x)][\bar{c}_b(x)\sigma_{\alpha\mu}\gamma_5 u_b(x)] - [\bar{d}_a(x)\sigma_{\alpha\mu}\gamma_5 c_a(x)][\bar{c}_b(x)\gamma^\alpha u_b(x)]\}, \quad (5)$$

$$J_\mu^{Z_c^1}(x) = \frac{1}{\sqrt{2}}\{[\bar{d}_a(x)\gamma_5 c_a(x)][\bar{c}_b(x)\gamma_\mu u_b(x)] + [\bar{d}_a(x)\gamma_\mu c_a(x)][\bar{c}_b(x)\gamma_5 u_b(x)]\}, \quad (6)$$

$$J_\mu^{Z_{cs}}(x) = \frac{1}{\sqrt{2}}\{[\bar{s}_a(x)\gamma_5 c_a(x)][\bar{c}_b(x)\gamma_\mu u_b(x)] + [\bar{s}_a(x)\gamma_\mu c_a(x)][\bar{c}_b(x)\gamma_5 u_b(x)]\}, \quad (7)$$

$$J_\mu^{Z_{cs}^1}(x) = \frac{1}{\sqrt{2}}\{[\bar{s}_a(x)\gamma^\alpha c_a(x)][\bar{c}_b(x)\sigma_{\alpha\mu}\gamma_5 u_b(x)] - [\bar{s}_a(x)\sigma_{\alpha\mu}\gamma_5 c_a(x)][\bar{c}_b(x)\gamma^\alpha u_b(x)]\}, \quad (8)$$

where $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu]$. The MDMs of the neutral Z_{cs} and Z_{cs}^1 states are also calculated. To do this, we substitute the d quark instead of the u quark in Eqs. (7) and (8). For the sake of brevity, only the numerical results of the neutral Z_{cs} states are presented. It is obvious that MDMs are zero for the neutral Z_c and Z_c^1 states.

The correlation function in Eq. (4) can be calculated in terms of quark and gluon degrees of freedom. To do this, we substitute interpolating currents given in Eqs. (5)–(8) into the correlation function and contract the corresponding quark fields via Wick's theorem. As a result, we get the following expressions for the $Z_{c(s)}$ states:

$$\Pi_{\mu\nu}^{\text{QCD}-Z_c}(p,q) = \frac{i}{2}\int d^4x e^{ipx}\langle 0|\{\text{Tr}[\gamma^\alpha S_c^{aa'}(x)\gamma^\beta S_d^{a'a}(-x)]\text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_u^{bb'}(x)\gamma_5\sigma_{\nu\beta}S_c^{b'b}(-x)]$$

$$- \text{Tr}[\gamma^\alpha S_c^{aa'}(x)\gamma_5\sigma_{\nu\beta}S_d^{a'a}(-x)]\text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_u^{bb'}(x)\gamma^\beta S_c^{b'b}(-x)]$$

$$- \text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_c^{aa'}(x)\gamma^\beta S_d^{a'a}(-x)]\text{Tr}[\gamma^\alpha S_u^{bb'}(x)\gamma_5\sigma_{\nu\beta}S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_c^{aa'}(x)\gamma_5\sigma_{\nu\beta}S_d^{a'a}(-x)]\text{Tr}[\gamma^\alpha S_u^{bb'}(x)\gamma^\beta S_c^{b'b}(-x)]\}|0\rangle_\gamma, \tag{9}$$

$$\Pi_{\mu\nu}^{\text{QCD}-Z_c^1}(p,q) = \frac{i}{2}\int d^4x e^{ipx}\langle 0|\{\text{Tr}[\gamma_5 S_c^{aa'}(x)\gamma_5 S_d^{a'a}(-x)]\text{Tr}[\gamma_\mu S_u^{bb'}(x)\gamma_\nu S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\gamma_5 S_c^{aa'}(x)\gamma_\nu S_d^{a'a}(-x)]\text{Tr}[\gamma_\mu S_u^{bb'}(x)\gamma_5 S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\gamma_\mu S_c^{aa'}(x)\gamma_5 S_d^{a'a}(-x)]\text{Tr}[\gamma_5 S_u^{bb'}(x)\gamma_\nu S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\gamma_\mu S_c^{aa'}(x)\gamma_\nu S_d^{a'a}(-x)]\text{Tr}[\gamma_5 S_u^{bb'}(x)\gamma_5 S_c^{b'b}(-x)]\}|0\rangle_\gamma, \tag{10}$$

$$\Pi_{\mu\nu}^{\text{QCD}-Z_{cs}}(p,q) = \frac{i}{2}\int d^4x e^{ipx}\langle 0|\{\text{Tr}[\gamma_5 S_c^{aa'}(x)\gamma_5 S_d^{a'a}(-x)]\text{Tr}[\gamma_\mu S_s^{bb'}(x)\gamma_\nu S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\gamma_5 S_c^{aa'}(x)\gamma_\nu S_d^{a'a}(-x)]\text{Tr}[\gamma_\mu S_s^{bb'}(x)\gamma_5 S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\gamma_\mu S_c^{aa'}(x)\gamma_5 S_d^{a'a}(-x)]\text{Tr}[\gamma_5 S_s^{bb'}(x)\gamma_\nu S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\gamma_\mu S_c^{aa'}(x)\gamma_\nu S_d^{a'a}(-x)]\text{Tr}[\gamma_5 S_s^{bb'}(x)\gamma_5 S_c^{b'b}(-x)]\}|0\rangle_\gamma, \tag{11}$$

$$\Pi_{\mu\nu}^{\text{QCD}-Z_{cs}^1}(p,q) = \frac{i}{2}\int d^4x e^{ipx}\langle 0|\{\text{Tr}[\gamma^\alpha S_c^{aa'}(x)\gamma^\beta S_u^{a'a}(-x)]\text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_s^{bb'}(x)\gamma_5\sigma_{\nu\beta}S_c^{b'b}(-x)]$$

$$- \text{Tr}[\gamma^\alpha S_c^{aa'}(x)\gamma_5\sigma_{\nu\beta}S_u^{a'a}(-x)]\text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_s^{bb'}(x)\gamma^\beta S_c^{b'b}(-x)]$$

$$- \text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_c^{aa'}(x)\gamma^\beta S_u^{a'a}(-x)]\text{Tr}[\gamma^\alpha S_s^{bb'}(x)\gamma_5\sigma_{\nu\beta}S_c^{b'b}(-x)]$$

$$+ \text{Tr}[\sigma_{\mu\alpha}\gamma_5 S_c^{aa'}(x)\gamma_5\sigma_{\nu\beta}S_u^{a'a}(-x)]\text{Tr}[\gamma^\alpha S_s^{bb'}(x)\gamma^\beta S_c^{b'b}(-x)]\}|0\rangle_\gamma, \tag{12}$$

where $S_{q(c)}(x)$ is propagator of the light q = u, d, s quarks and c quark, and it is identified as

$$S_q(x) = \frac{1}{2\pi^2 x^2}\left(i\frac{\not{x}}{x^2} - \frac{m_q}{2}\right) - \frac{\langle\bar{q}q\rangle}{12}\left(1 - i\frac{m_q\not{x}}{4}\right) - \frac{\langle\bar{q}\sigma.Gq\rangle}{192}x^2\left(1 - i\frac{m_q\not{x}}{6}\right) - \frac{ig_s}{32\pi^2 x^2}G^{\mu\nu}(x)[\not{x}\sigma_{\mu\nu} + \sigma_{\mu\nu}\not{x}], \tag{13}$$

$$S_c(x) = \frac{m_c^2}{4\pi^2}\left[\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}} + i\frac{\not{x}K_2(m_c\sqrt{-x^2})}{(\sqrt{-x^2})^2}\right] - \frac{g_s m_c}{16\pi^2}\int_0^1 dv G^{\mu\nu}(vx)\left[(\sigma_{\mu\nu}\not{x} + \not{x}\sigma_{\mu\nu})\frac{K_1(m_c\sqrt{-x^2})}{\sqrt{-x^2}}\right.$$

$$\left. + 2\sigma_{\mu\nu}K_0(m_c\sqrt{-x^2})\right]. \tag{14}$$

The correlation functions in Eqs. (9)–(12) include various contributions: the photon can be emitted perturbatively and nonperturbatively. The next step is to calculate these contributions. These procedures are standard in the LCSR but quite lengthy. Therefore, we do not present these steps. The interested readers can find details of such calculations in Ref. [60]. With these calculations, the results of the QCD representation were obtained. We can move on to calculate the results for the hadronic representation.

The correlation function in Eq. (4) can also be rewritten by inserting the a complete set of the $Z_{c(s)}$ tetraquark currents with the same quantum numbers into Eq. (5) as follows:

$$\Pi_{\mu\nu}^{Had}(p,q) = \frac{\langle 0|J_\mu(x)|Z_{c(s)}(p)\rangle}{p^2 - m_{Z_{c(s)}}^2}\langle Z_{c(s)}(p)|Z_{c(s)}(p+q)\rangle_\gamma \frac{\langle Z_{c(s)}(p+q)|J_\nu^\dagger(0)|0\rangle}{(p+q)^2 - m_{Z_{c(s)}}^2} + \cdots, \tag{15}$$

where dots stands for the contributions coming from the higher states and continuum. In the existence of the external electromagnetic background field, the matrix element $\langle Z_{c(s)}(p)|Z_{c(s)}(p+q)\rangle_\gamma$ can be expressed associated with the Lorentz invariant form factors as follows [61]:

$$\langle Z_{c(s)}(p)|Z_{c(s)}(p+q)\rangle_\gamma = -\varepsilon^\tau(\varepsilon^\theta)^\alpha(\varepsilon^\delta)^\beta \Big[G_1(Q^2)(2p+q)_\tau g_{\alpha\beta} + G_2(Q^2)(g_{\tau\beta}q_\alpha - g_{\tau\alpha}q_\beta)$$

$$- \frac{G_3(Q^2)}{2m_{Z_{c(s)}}^2}(2p+q)_\tau q_\alpha q_\beta \Big], \tag{16}$$

where ε^δ and ε^θ are the polarization vectors of the initial and final $Z_{c(s)}$ states, and q and ε^τ are the momentum and polarization vector of the photon, respectively.

The magnetic form factors $F_M(Q^2)$ can be written in the following way:

$$F_M(Q^2) = G_2(Q^2), \tag{17}$$

where $Q^2 = -q^2$. At zero momentum transfer, i.e., $Q^2 = 0$, the form factors $F_M(0)$, are proportional to the magnetic moment $\mu_{Z_{c(s)}}$ in the following way:

$$\mu_{Z_{c(s)}} = \frac{e}{2m_{Z_{c(s)}}} F_M(0). \tag{18}$$

Using Eqs. (15) and (16), the correlation function takes the form of

$$\Pi_{\mu\nu}^{Had}(p,q) = \frac{\lambda_{Z_{c(s)}}^2}{[m_{Z_{c(s)}}^2 - (p+q)^2][m_{Z_{c(s)}}^2 - p^2]} \Big[F_M(0)\Big(q_\mu\varepsilon_\nu - q_\nu\varepsilon_\mu + \frac{\varepsilon.p}{m_{Z_{c(s)}}^2}(p_\mu q_\nu - p_\nu q_\mu)\Big)$$

$$+ \text{ other independent form factors} \Big]. \tag{19}$$

The final step for analytical calculations is to match the results obtained using two different representations. To do this, appropriate Lorentz structures are selected, and Borel transform and continuum subtraction are applied to eliminate the contribution of higher states and continuum terms. Finally, we choose the structure $q_\mu\varepsilon_\nu$ for the MDMs, and we get

$$\mu_{Z_c}\lambda_{Z_c}^2 = e^{\frac{m_{Z_c}^2}{M^2}}\Delta_1^{QCD}, \tag{20}$$

$$\mu_{Z_c^1}\lambda_{Z_c^1}^2 = e^{\frac{m_{Z_c^1}^2}{M^2}}\Delta_2^{QCD}, \tag{21}$$

$$\mu_{Z_{cs}}\lambda_{Z_{cs}}^2 = e^{\frac{m_{Z_{cs}}^2}{M^2}}\Delta_3^{QCD}, \tag{22}$$

$$\mu_{Z_{cs}^1}\lambda_{Z_{cs}^1}^2 = e^{\frac{m_{Z_{cs}^1}^2}{M^2}}\Delta_4^{QCD}. \tag{23}$$

The explicit expressions of the Δ_i^{QCD} functions are presented in the Appendix.

III. NUMERICAL ANALYSIS

In this section, we numerically analyze the result of calculations for the MDMs. We use $m_u = m_d = 0$,

$m_s = 96^{+8.0}_{-4.0}$ MeV, $m_c = (1.275 \pm 0.025)$ GeV, $m_{Z_c} = 4022.9 \pm 0.8 \pm 2.7$ MeV, $m_{Z_c^1} = 4196^{+31+17}_{-29-13}$ MeV, $m_{Z_{cs}} = 4003.0 \pm 6.0^{+4.0}_{-14.0}$ MeV, $m_{Z_{cs}^1} = 4216.0 \pm 24.0^{+43}_{-30}$ MeV, $f_{3\gamma} = -0.0039$ GeV^2 [62], $\chi = -2.85 \pm 0.5$ GeV^{-2} [63] $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$, $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = (-0.24 \pm 0.01)^3$ GeV^3 [64], $m_0^2 = 0.8 \pm 0.1$ GeV^2, $\langle g_s^2 G^2\rangle = 0.88$ GeV^4 [20]. To proceed with calculations, we need numerical values of the residues of the $Z_{c(s)}$ states. They are borrowed from Refs. [25,50], which were obtained in the framework of the two-point QCD sum rules. The parameters used in the photon distribution amplitudes are given in Ref. [62].

Besides these input parameters used, the expressions for the MDMs contain two auxiliary parameters: continuum threshold s_0 and Borel mass parameter M^2. The standard prescription in LCSR is that physical observables should be weakly dependent on auxiliary parameters. In other words, we need to find the working regions of these auxiliary parameters, on which MDMs are independent. To obtain the working regions of these helping parameters, the standard ways of the LCSR approach, such as weak dependence of the results on the helping parameters, convergence of the OPE and pole dominance, are considered. Our numerical calculations lead to the conclusion that these constraints are satisfied in the intervals shown below for the considered tetraquark states:

$18.6 \text{ GeV}^2 \leq s_0 \leq 20.6 \text{ GeV}^2$ for Z_c state,

$4.5 \text{ GeV}^2 \leq M^2 \leq 6.5 \text{ GeV}^2$ for Z_c state,

$20.3 \text{ GeV}^2 \leq s_0 \leq 22.1 \text{ GeV}^2$ for Z_c^1 state,

$5.0 \text{ GeV}^2 \leq M^2 \leq 7.0 \text{ GeV}^2$ for Z_c^1 state,

$18.3 \text{ GeV}^2 \leq s_0 \leq 20.1 \text{ GeV}^2$ for Z_{cs} state,

$4.0 \text{ GeV}^2 \leq M^2 \leq 6.0 \text{ GeV}^2$ for Z_{cs} state,

$20.5 \text{ GeV}^2 \leq s_0 \leq 22.5 \text{ GeV}^2$ for Z_{cs}^1 state,

$5.0 \text{ GeV}^2 \leq M^2 \leq 7.0 \text{ GeV}^2$ for Z_{cs}^1 state.

Since we have numerical values of all input parameters, we can start numerical calculations. In Fig. 1, we plot the dependencies of the MDMs on M^2 at three values of s_0. As is seen, the MDMs of $Z_{c(s)}$ states shows good stability with respect to M^2 in its working window. While MDMs shows some dependence on s_0, it remains inside the limits allowed by the LCSR and constitutes the major parts of the ambiguities.

Our final results for the MDMs of charged $Z_{c(s)}$ states are

$$\mu_{Z_c} = 0.66^{+0.27}_{-0.25}\mu_N, \tag{24}$$

$$\mu_{Z_c^1} = 1.03^{+0.32}_{-0.29}\mu_N, \tag{25}$$

$$\mu_{Z_{cs}} = 0.73^{+0.28}_{-0.26}\mu_N, \tag{26}$$

$$\mu_{Z_{cs}^1} = 0.77^{+0.27}_{-0.25}\mu_N. \tag{27}$$

The uncertainties in the results are due to s_0, M^2 and other input parameters. We should note that the main source of uncertainties is s_0. It is seen that the results obtained for these states are large enough to be measured experimentally. Unfortunately, there are no experimental data and theoretical results that we can compare.

In order to obtain a more profound comprehension of the underlying quark-gluon dynamics, it is useful to consider the individual quark sector contributions to the MDMs. This can be achieved by dialing the corresponding charge factors e_u, e_d, e_s and e_c. When it was done in the cases of
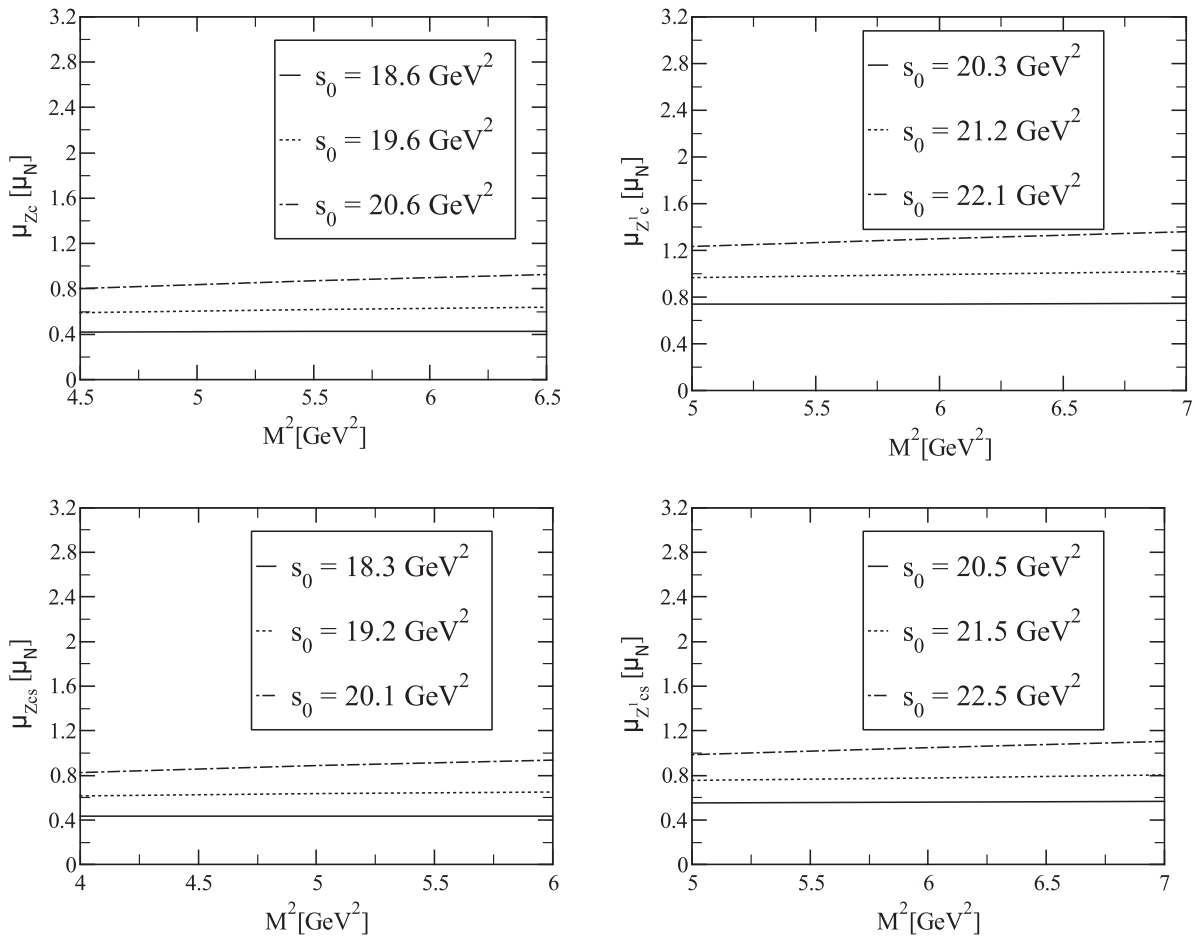

FIG. 1. The dependencies of MDMs of the μ_{Z_c}, $\mu_{Z_c^1}$, $\mu_{Z_{cs}}$ and $\mu_{Z_{cs}^1}$ states on M^2 at three different values of s_0.

Z_c and Z_c^1 states, we see that terms proportional the e_u contributes about 67% to the total MDMs, e_d about 33%, and e_c almost zero. In the cases of Z_{cs} and Z_{cs}^1 states, we observe that terms proportional the e_u contributes about 69% to the total MDMs, e_s about 31%, and e_c almost zero. A closer investigation shows that the smallness of the e_c contribution is because of an almost exact cancellation of the terms involving the e_c.

As we mentioned in the Introduction, the $Z_{cs}(3985)$ and Z_{cs} states have very different decay widths and can be considered as two different states, in spite of being very close in mass. Also, there is a possibility that $Z_{cs}(3985)$ and Z_{cs} states are identical, but their masses and decay widths may not be consistent because of coupled-channels effects or experimental resolution. In the literature, there are few attempts to have some assignments on the substructure of Z_{cs} and Z_{cs}^1 states. In Ref. [29], they showed that the $Z_{cs}(3985)$ state is the U-spin partner of $Z_c(3900)$ as a resonance within the coupled-channel calculation in the $SU(3)_f$ symmetry and heavy quark spin symmetry. They predicted a new Z_{cs} state with a mass ∼4130 MeV, which is the U/V-spin partner state of charged $Z_c(4020)$ and heavy quark spin symmetry partner state of $Z_{cs}(3985)$. In Ref. [48], structures of the Z_c and $Z_{cs}(3985)$ states have been obtained using the chiral constituent quark model in a coupled-channels calculation. They obtained that $Z_{cs}(3985)$ and $Z_{cs}(4000)$ are the same state. They also predicted a new state, the $SU(3)_f$ partner of the $Z_c(4020)$ state at ∼4110 MeV. In Ref. [36], the mass spectrum of the $Z_{cs}(3985)$ state has been obtained in the framework of the chiral perturbation theory. They predicted a new state with a mass around ∼4125 MeV, which can be a strange partner of the $Z_c(4020)$ state. In Ref. [50], mass, decay constants and production rates of the $Z_{c(s)}$ states were investigated in a molecular picture within the QCD sum rules. The obtained results supported that $Z_{cs}(3985)$, $Z_{cs}(4000)$ and $Z_{cs}(4220)$ were strange partners of $X(3872)$, $Z_c(3900)$ and $Z_c(4020)$, respectively. In Ref. [51], they studied the properties of the $Z_{cs}(3985)$ and $Z_{cs}(4000)$ states in the molecular picture via solvable nonrelativistic effective field theory. They found that $Z_{cs}(3985)$ and $Z_{cs}(4000)$ states are different states, and the $Z_{cs}(4220)$ state is a heavy quark spin symmetry partner state of $Z_{cs}(4000)$. Also, they predicted that $Z_{cs}(4000)$ and $Z_{cs}(4220)$ states are the $SU(3)_f$ partners of the $Z_c(3900)$ and $Z_c(4020)$ states, respectively. In Ref. [55], the MDMs of the $Z_{cs}(3985)$ state have been acquired in the framework of LCSR by using the molecular and compact diquark-antidiquark type interpolating currents. The obtained results are given as $\mu_{Z_{cs}}^{Di} = 0.60^{+0.26}_{-0.21}\mu_N$ and $\mu_{Z_{cs}}^{Mol} = 0.52^{+0.19}_{-0.17}\mu_N$ for diquark-antidiquark and molecular pictures, respectively. When we compare our results, it can be seen that the difference between the central values is large enough to say that these two states are different states. Therefore, our MDM results

support that $Z_{cs}(3985)$ and Z_{cs} are different states. We also mentioned that the Z_{cs}^1 state can be interpreted as the strange partner of the Z_c state. We predict that if the Z_c and Z_{cs}^1 states are $SU(3)_f$ partners, the difference between their MDMs should be around 15%. If this difference is more than 15%, these states are not $SU(3)_f$ partners. As can be seen from studies of the Z_c and Z_{cs} states, the results achieved using different approaches give rise to different estimations. The properties of these states should be tested in more precise experimental data in the future, and we need more experimental and theoretical studies on the different decay channels of $Z_{c(s)}$ to figure out their inner configurations.

Our final remark is that the MDMs of the neutral Z_{cs} and Z_{cs}^1 states have also been extracted. The results are presented as

$$\mu_{Z_{cs}^0} = (3.0^{+0.9}_{-0.8}) \times 10^{-2}\mu_N, \tag{28}$$

$$\mu_{Z_{cs}^{1.0}} = (3.6^{+0.9}_{-0.9}) \times 10^{-2}\mu_N. \tag{29}$$

The MDMs of the neutral Z_{cs} and Z_{cs}^1 states are obtained as nonzero but small values. The nonzero values of the MDMs of neutral Z_{cs} and Z_{cs}^1 states are due to the $SU(3)_f$ symmetry breaking.

IV. DISCUSSION AND CONCLUDING REMARKS

To investigate the hidden-charm tetraquark states with and without strangeness, we have acquired the magnetic dipole moments for the molecular picture with $J^P = 1^+$ quantum numbers within light-cone QCD sum rules. For this aim, we use the relevant molecular interpolating currents and the photon distribution amplitudes. The electromagnetic form factors of particles encode the spatial distributions of charge and magnetization in the particles and ensure significant knowledge about the quark organizations of the particles and the underlying dynamics. The size of the results on $Z_c(4020)^+$, $Z_c(4200)^+$, $Z_{cs}(4000)^+$ and $Z_{cs}(4220)^+$ states indicates that the magnetic dipole moments of these states are accessible in the experiment. The magnetic dipole moments of the neutral $Z_{cs}(4000)$ and $Z_{cs}(4220)$ states are also nonzero; however, their values are small. The direct measurement of the magnetic dipole moments of exotic states are not likely in the near future. For this reason, any indirect predictions of the magnetic dipole moments of the exotic states would be very helpful. Any experimental measurements of the magnetic dipole moments of the hidden-charm tetraquark states and comparison of the acquired results with the estimations of this work may ensure useful information on the substructure of the these tetraquark states as well as the nonperturbative behaviors of the strong interaction at the low-energy region.

ACKNOWLEDGMENTS

APPENDIX: EXPLICIT FORMS OF THE Δ_i^{QCD} FUNCTIONS

In this Appendix, we give the explicit expressions for the Δ_i^{QCD} functions:

$$
\begin{aligned}
\Delta_1^{\mathrm{QCD}} = \frac{1}{47185920\pi^5} \{ & 4e_c m_c m_q (20P_1(7I[0,3,1,0] - 20I[0,3,1,1] + 19I[0,3,1,2] - 6I[0,3,1,3] - 14I[0,3,2,0] \\
& + 26I[0,3,2,1] - 12I[0,3,2,2] + 7I[0,3,3,0] - 6I[0,3,3,1] + 18I[1,2,1,1] - 36I[1,2,1,2] + 18I[1,2,1,3] \\
& - 36I[1,2,2,1] + 36I[1,2,2,2] + 18I[1,2,3,1]) + 27(6I[0,5,1,1] - 18I[0,5,1,2] + 18I[0,5,1,3] - 6I[0,5,1,4] \\
& - 16I[0,5,2,1] + 31I[0,5,2,2] - 14I[0,5,2,3] - I[0,5,2,4] + 14I[0,5,3,1] - 12I[0,5,3,2] - 2I[0,5,3,3] \\
& - 4I[0,5,4,1] - I[0,5,4,2] + 5I[1,4,2,2] - 10I[1,4,2,3] + 5I[1,4,2,4] - 10I[1,4,3,2] + 10I[1,4,3,3] \\
& + 5I[1,4,4,2])) \\
& - e_d(-15P_1(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2] \\
& + 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 12I[1,3,2,1] - 12I[1,3,2,2] + 4I[1,3,2,3] - 24I[1,3,3,1] \\
& + 12I[1,3,3,2] + 12I[1,3,4,1]) + 6480\pi^2 m_c P_2(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] \\
& - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2] + 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 12I[1,3,2,1] \\
& - 12I[1,3,2,2] + 4I[1,3,2,3] - 24I[1,3,3,1] + 12I[1,3,3,2] + 12I[1,3,4,1]) + 8m_c^2(20P_1(I[0,3,1,0] \\
& - 3I[0,3,1,1] + 3I[0,3,1,2] - I[0,3,1,3] - 2I[0,3,2,0] + 4I[0,3,2,1] - 2I[0,3,2,2] + I[0,3,3,0] \\
& - I[0,3,3,1] + 3I[1,2,1,1] - 6I[1,2,1,2] + 3I[1,2,1,3] - 6I[1,2,2,1] + 6I[1,2,2,2] + 3I[1,2,3,1]) \\
& + 27(2I[0,5,2,1] - 5I[0,5,2,2] + 4I[0,5,2,3] - I[0,5,2,4] - 4I[0,5,3,1] + 6I[0,5,3,2] - 2I[0,5,3,3] \\
& + 2I[0,5,4,1] - I[0,5,4,2] + 5I[1,4,2,2] - 10I[1,4,2,3] + 5I[1,4,2,4] - 10I[1,4,3,2] + 10I[1,4,3,3] \\
& + 5I[1,4,4,2])) + 243(I[0,6,3,0] - 4I[0,6,3,1] + 6I[0,6,3,2] - 4I[0,6,3,3] + I[0,6,3,4] - 3I[0,6,4,0] \\
& + 9I[0,6,4,1] - 9I[0,6,4,2] + 3I[0,6,4,3] + 3I[0,6,5,0] - 6I[0,6,5,1] + 3I[0,6,5,2] - I[0,6,6,0] + I[0,6,6,1] \\
& + 6I[1,5,3,1] - 18I[1,5,3,2] + 18I[1,5,3,3] - 6I[1,5,3,4] - 18I[1,5,4,1] + 36I[1,5,4,2] - 18I[1,5,4,3] \\
& + 18I[1,5,5,1] - 18I[1,5,5,2] - 6I[1,5,6,1])) \\
& + e_u(-15P_1(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2] \\
& + 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 12I[1,3,2,1] - 12I[1,3,2,2] + 4I[1,3,2,3] - 24I[1,3,3,1] \\
& + 12I[1,3,3,2] + 12I[1,3,4,1]) + 8m_c^2(-720\pi^2 m_d P_2(I[0,3,2,0] - 2I[0,3,2,1] + I[0,3,2,2] - 2I[0,3,3,0] \\
& + 2I[0,3,3,1] + I[0,3,4,0] + 6I[1,2,2,1] - 3I[1,2,2,2] - 6I[1,2,3,1]) + 20P_1(I[0,3,1,0] - 3I[0,3,1,1] \\
& + 3I[0,3,1,2] - I[0,3,1,3] - 2I[0,3,2,0] + 4I[0,3,2,1] - 2I[0,3,2,2] + I[0,3,3,0] - I[0,3,3,1] + 3I[1,2,1,1] \\
& - 6I[1,2,1,2] + 3I[1,2,1,3] - 6I[1,2,2,1] + 6I[1,2,2,2] + 3I[1,2,3,1]) + 27(2I[0,5,2,1] - 5I[0,5,2,2] \\
& + 4I[0,5,2,3] - I[0,5,2,4] - 4I[0,5,3,1] + 6I[0,5,3,2] - 2I[0,5,3,3] + 2I[0,5,4,1] - I[0,5,4,2] \\
& + 5I[1,4,2,2] - 10I[1,4,2,3] + 5I[1,4,2,4] - 10I[1,4,3,2] + 10I[1,4,3,3] + 5I[1,4,4,2])) \\
& + 324m_c(20\pi^2 P_2(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] \\
& - 3I[0,4,3,2] + 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 12I[1,3,2,1] - 12I[1,3,2,2] + 4I[1,3,2,3]
\end{aligned}
$$

$$-24I[1,3,3,1]+12I[1,3,3,2]+12I[1,3,4,1])+3m_q(I[0,5,2,0]-4I[0,5,2,1]+6I[0,5,2,2]-4I[0,5,2,3]$$

$$+I[0,5,2,4]-3I[0,5,3,0]+9I[0,5,3,1]-9I[0,5,3,2]+3I[0,5,3,3]+3I[0,5,4,0]-6I[0,5,4,1]+3I[0,5,4,2]$$

$$-I[0,5,5,0]+I[0,5,5,1]+5I[1,4,2,1]-15I[1,4,2,2]+15I[1,4,2,3]-5I[1,4,2,4]-15I[1,4,3,1]$$

$$+30I[1,4,3,2]-15I[1,4,3,3]+15I[1,4,4,1]-15I[1,4,4,2]-5I[1,4,5,1]))+243(I[0,6,3,0]$$

$$-4I[0,6,3,1]+6I[0,6,3,2]-4I[0,6,3,3]+I[0,6,3,4]-3I[0,6,4,0]+9I[0,6,4,1]-9I[0,6,4,2]$$

$$+3I[0,6,4,3]+3I[0,6,5,0]-6I[0,6,5,1]+3I[0,6,5,2]-I[0,6,6,0]+I[0,6,6,1]+6I[1,5,3,1]$$

$$-18I[1,5,3,2]+18I[1,5,3,3]-6I[1,5,3,4]-18I[1,5,4,1]+36I[1,5,4,2]-18I[1,5,4,3]+18I[1,5,5,1]$$

$$-18I[1,5,5,2]-6I[1,5,6,1]))+20\pi^2 f_{3\gamma}P_2 m_q(4(32e_d+41e_u)I[0,3,4,0]+3(9e_d+20e_u)I[0,3,5,0])I_2[\mathcal{V}]$$

$$+3m_0^2(e_d+14e_u)I_4[\tilde{\mathcal{S}}]I[0,5,4,0]\},\tag{A1}$$

$$\Delta_2^{\text{QCD}}=\frac{1}{10485760\pi^5}\{160\pi^2 e_c m_c P_2(-3I[0,4,1,0]+9I[0,4,1,1]-9I[0,4,1,2]+3I[0,4,1,3]+10I[0,4,2,0]$$

$$-21I[0,4,2,1]+12I[0,4,2,2]-I[0,4,2,3]-11I[0,4,3,0]+13I[0,4,3,1]-2I[0,4,3,2]+4I[0,4,4,0]$$

$$-I[0,4,4,1]+m_0^2(I[0,3,2,0]-2I[0,3,2,1]+I[0,3,2,2]-2I[0,3,3,0]+2I[0,3,3,1]$$

$$+I[0,3,4,0]+6I[1,2,2,1]-3I[1,2,2,2]-6I[1,2,3,1])+4I[1,3,2,1]-8I[1,3,2,2]+4I[1,3,2,3]$$

$$-8I[1,3,3,1]+8I[1,3,3,2]+4I[1,3,4,1])$$

$$+e_u(5P_1(I[0,4,2,0]-3I[0,4,2,1]+3I[0,4,2,2]-I[0,4,2,3]-3I[0,4,3,0]+6I[0,4,3,1]-3I[0,4,3,2]$$

$$+3I[0,4,4,0]-3I[0,4,4,1]-I[0,4,5,0]+12I[1,3,2,1]-12I[1,3,2,2]+4I[1,3,2,3]-24I[1,3,3,1]$$

$$+12I[1,3,3,2]+12I[1,3,4,1])-240\pi^2 m_c P_2(I[0,4,2,0]-3I[0,4,2,1]+3I[0,4,2,2]-I[0,4,2,3]$$

$$-3I[0,4,3,0]+6I[0,4,3,1]-3I[0,4,3,2]+3I[0,4,4,0]-3I[0,4,4,1]-I[0,4,5,0]+12I[1,3,2,1]$$

$$-12I[1,3,2,2]+4I[1,3,2,3]-24I[1,3,3,1]+12I[1,3,3,2]+12I[1,3,4,1])-9(I[0,6,3,0]-4I[0,6,3,1]$$

$$+6I[0,6,3,2]-4I[0,6,3,3]+I[0,6,3,4]-3I[0,6,4,0]+9I[0,6,4,1]-9I[0,6,4,2]+3I[0,6,4,3]$$

$$+3I[0,6,5,0]-6I[0,6,5,1]+3I[0,6,5,2]-I[0,6,6,0]+I[0,6,6,1]+6I[1,5,3,1]-18I[1,5,3,2]$$

$$+18I[1,5,3,3]-6I[1,5,3,4]-18I[1,5,4,1]+36I[1,5,4,2]-18I[1,5,4,3]+18I[1,5,5,1]-18I[1,5,5,2]$$

$$-6I[1,5,6,1]))$$

$$+e_d(-5P_1(I[0,4,2,0]-3I[0,4,2,1]+3I[0,4,2,2]-I[0,4,2,3]-3I[0,4,3,0]+6I[0,4,3,1]-3I[0,4,3,2]$$

$$+3I[0,4,4,0]-3I[0,4,4,1]-I[0,4,5,0]+12I[1,3,2,1]-12I[1,3,2,2]+4I[1,3,2,3]-24I[1,3,3,1]$$

$$+12I[1,3,3,2]+12I[1,3,4,1])+240\pi^2 m_c P_2(I[0,4,2,0]-3I[0,4,2,1]+3I[0,4,2,2]-I[0,4,2,3]$$

$$-3I[0,4,3,0]+6I[0,4,3,1]-3I[0,4,3,2]+3I[0,4,4,0]-3I[0,4,4,1]-I[0,4,5,0]+12I[1,3,2,1]$$

$$-12I[1,3,2,2]+4I[1,3,2,3]-24I[1,3,3,1]+12I[1,3,3,2]+12I[1,3,4,1])+9(I[0,6,3,0]-4I[0,6,3,1]$$

$$+6I[0,6,3,2]-4I[0,6,3,3]+I[0,6,3,4]-3I[0,6,4,0]+9I[0,6,4,1]-9I[0,6,4,2]+3I[0,6,4,3]$$

$$+3I[0,6,5,0]-6I[0,6,5,1]+3I[0,6,5,2]-I[0,6,6,0]+I[0,6,6,1]+6I[1,5,3,1]-18I[1,5,3,2]$$

$$+18I[1,5,3,3]-6I[1,5,3,4]-18I[1,5,4,1]+36I[1,5,4,2]-18I[1,5,4,3]+18I[1,5,5,1]-18I[1,5,5,2]$$

$$-6I[1,5,6,1]))+P_1[(6e_u-7e_d)(I[0,4,3,1]-3I[0,4,4,0]+3I[0,4,5,2]-I[0,4,6,2]+4I[1,3,3,1]$$

$$-4I[1,3,6,2]+2I[2,2,3,3]-6I[2,2,4,2]+6I[2,2,5,0]-2I[2,2,6,4])\mathbb{A}[u_0]-5(11e_u+13e_d)\{(I[0,4,2,0]$$

$$-I[0,4,5,1]+2I[1,3,2,1]-6I[1,3,3,2]+6I[1,3,4,2])(I_2[\mathcal{S}]-2I_2[\mathcal{T}_2]-I_2[\mathcal{T}_3]+I_2[\mathcal{T}_4])+(I[0,4,2,3]$$

$$-3I[0,4,3,0]+3I[0,4,4,1]-I[0,4,5,1])(I_2[\mathcal{T}_1]+I_2[\tilde{\mathcal{S}}])\}+15(5e_u-11e_d)(I[0,4,3,1]-3I[0,4,4,1]$$

$$+2I[1,3,3,2]-6I[1,3,4,1]+6I[1,3,5,1]-2I[1,3,6,1])I_3[h_\gamma]-3\chi(2I[0,5,3,2]-6I[0,5,4,2]+6I[0,5,5,2]$$

$$-2I[0,5,6,2]+5I[1,4,3,2]-15I[1,4,4,1]+15I[1,4,5,0]-5I[1,4,6,0])\varphi_\gamma[u_0]\},\tag{A2}$$

$$\Delta_3^{\text{QCD}} = \frac{1}{283115520\pi^5}\{16e_Q m_c(-40m_c m_s P_1 P_3 \pi^2(I[0,1,1,0] - I[0,1,1,1] - I[0,1,2,0])$$

$$+ 360\pi^2(P_2 - P_3)(-3I[0,4,1,0] + 9II[0,4,1,1] - 9I[0,4,1,2] + 3I[0,4,1,3] + 10I[0,4,2,0] - 21I[0,4,2,1]$$

$$+ 12I[0,4,2,2] - I[0,4,2,3] - 11I[0,4,3,0] + 13I[0,4,3,1] - 2I[0,4,3,2] + 4I[0,4,4,0] - I[0,4,4,1]$$

$$+ m_0^2(I[0,3,2,0] - 2I[0,3,2,1] + I[0,3,2,2] - 2I[0,3,3,0] + 2I[0,3,3,1] + I[0,3,4,0] + 6I[1,2,2,1]$$

$$- 3I[1,2,2,2] - 6I[1,2,3,1]) + 4I[1,3,2,1] - 8I[1,3,2,2] + 4I[1,3,2,3] - 8I[1,3,3,1] + 8I[1,3,3,2]$$

$$+ 4I[1,3,4,1]) + m_s(35P_1(I[0,3,1,0] - 2I[0,3,1,1] + I[0,3,1,2] - 2I[0,3,2,0] + 2I[0,3,2,1] + I[0,3,3,0])$$

$$- 27(6I[0,5,1,1] - 18I[0,5,1,2] + 18I[0,5,1,3] - 6I[0,5,1,4] - 16I[0,5,2,1] + 31I[0,5,2,2] - 14I[0,5,2,3]$$

$$- I[0,5,2,4] + 14I[0,5,3,1] - 12I[0,5,3,2] - 2I[0,5,3,3] - 4I[0,5,4,1] - I[0,5,4,2] + 5I[1,4,2,2]$$

$$- 10I[1,4,2,3] + 5I[1,4,2,4] - 10I[1,4,3,2] + 10I[1,4,3,3] + 5I[1,4,4,2])))$$

$$+ e_s(-160m_c^2 P_1(I[0,3,1,0] + I[0,3,1,1] - 3I[0,3,1,2] + I[0,3,1,3] - 2I[0,3,2,0] + I[0,3,3,0] - I[0,3,3,1]$$

$$+ 3I[1,2,1,1] - 3I[1,2,1,3] - 6I[1,2,2,1] + 3I[1,2,3,1]) + 5P_1(25I[0,4,2,0] - 75I[0,4,2,1] + 75I[0,4,2,2]$$

$$- 25I[0,4,2,3] - 71I[0,4,3,0] + 142I[0,4,3,1] - 71I[0,4,3,2] + 67I[0,4,4,0] - 67I[0,4,4,1] - 21I[0,4,5,0]$$

$$+ 268I[1,3,2,1] - 284I[1,3,2,2] + 100I[1,3,2,3] - 536I[1,3,3,1] + 284I[1,3,3,2] + 268I[1,3,4,1])$$

$$+ 960\pi^2 m_c P_2(2P_1(I[0,2,1,0] - 2I[0,2,1,1] + I[0,2,1,2] - 2I[0,2,2,0] + 2I[0,2,2,1] + I[0,2,3,0]$$

$$- 2I[1,1,1,0] + 4I[1,1,1,1] - 2I[1,1,1,2] + 4I[1,1,2,0] - 4I[1,1,2,1] - 2I[1,1,3,0]) - 9(I[0,4,2,0]$$

$$- 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2] + 3I[0,4,4,0]$$

$$- 3I[0,4,4,1] - I[0,4,5,0] + 12I[1,3,2,1] - 12I[1,3,2,2] + 4I[1,3,2,3] - 24I[1,3,3,1] + 12I[1,3,3,2]$$

$$+ 12I[1,3,4,1])) - 324(I[0,6,3,0] - 4I[0,6,3,1] + 6I[0,6,3,2] - 4I[0,6,3,3] + I[0,6,3,4] - 3I[0,6,4,0]$$

$$+ 9I[0,6,4,1] - 9I[0,6,4,2] + 3I[0,6,4,3] + 3I[0,6,5,0] - 6I[0,6,5,1] + 3I[0,6,5,2] - I[0,6,6,0] + I[0,6,6,1]$$

$$+ 6I[1,5,3,1] - 18I[1,5,3,2] + 18I[1,5,3,3] - 6I[1,5,3,4] - 18I[1,5,4,1] + 36I[1,5,4,2] - 18I[1,5,4,3]$$

$$+ 18I[1,5,5,1] - 18I[1,5,5,2] - 6I[1,5,6,1]))$$

$$+ e_u(-160m_c^2 P_1(16\pi^2 m_s P_3(I[0,1,1,0] - I[0,1,1,1] - I[0,1,2,0]) - I[0,3,1,0] - I[0,3,1,1] + 3I[0,3,1,2]$$

$$- I[0,3,1,3] + 2I[0,3,2,0] - I[0,3,3,0] + I[0,3,3,1] - 3I[1,2,1,1] + 3I[1,2,1,3] + 6I[1,2,2,1] - 3I[1,2,3,1])$$

$$- 5P_1(25I[0,4,2,0] - 75I[0,4,2,1] + 75I[0,4,2,2] - 25I[0,4,2,3] - 71I[0,4,3,0] + 142I[0,4,3,1]$$

$$- 71I[0,4,3,2] + 67I[0,4,4,0] - 67I[0,4,4,1] - 21I[0,4,5,0] + 268I[1,3,2,1] - 284I[1,3,2,2] + 100I[1,3,2,3]$$

$$- 536I[1,3,3,1] + 284I[1,3,3,2] + 268I[1,3,4,1]) + 48m_c(-10P_1(4P_3\pi^2(I[0,2,1,0] - 2I[0,2,1,1]$$

$$+ I[0,2,1,2] - 2I[0,2,2,0] + 2I[0,2,2,1] + I[0,2,3,0] - 2I[1,1,1,0] + 4I[1,1,1,1] - 2I[1,1,1,2] + 4I[1,1,2,0]$$

$$- 4I[1,1,2,1] - 2I[1,1,3,0]) + m_s(I[0,3,1,0] - 3I[0,3,1,1] + 3I[0,3,1,2] - I[0,3,1,3] - 2I[0,3,2,0]$$

$$+ 4I[0,3,2,1] - 2I[0,3,2,2] + I[0,3,3,0] - I[0,3,3,1] + 3I[1,2,1,1] - 6I[1,2,1,2] + 3I[1,2,1,3] - 6I[1,2,2,1]$$

$$+ 6I[1,2,2,2] + 3I[1,2,3,1])) + 9(20\pi^2 P_3(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0]$$

$$+ 6I[0,4,3,1] - 3I[0,4,3,2] + 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 12I[1,3,2,1] - 12I[1,3,2,2]$$

$$+ 4I[1,3,2,3] - 24I[1,3,3,1] + 12I[1,3,3,2] + 12I[1,3,4,1]) + 3m_s(I[0,5,2,0] - 4I[0,5,2,1] + 6I[0,5,2,2]$$

$$- 4I[0,5,2,3] + I[0,5,2,4] - 3I[0,5,3,0] + 9I[0,5,3,1] - 9I[0,5,3,2] + 3I[0,5,3,3] + 3I[0,5,4,0]$$

$$- 6I[0,5,4,1] + 3I[0,5,4,2] - I[0,5,5,0] + I[0,5,5,1] + 5I[1,4,2,1] - 15I[1,4,2,2] + 15I[1,4,2,3]$$

$$- 5I[1,4,2,4] - 15I[1,4,3,1] + 30I[1,4,3,2] - 15I[1,4,3,3] + 15I[1,4,4,1] - 15I[1,4,4,2] - 5I[1,4,5,1])))$$

$$+ 324(I[0,6,3,0] - 4I[0,6,3,1] + 6I[0,6,3,2] - 4I[0,6,3,3] + I[0,6,3,4] - 3I[0,6,4,0] + 9I[0,6,4,1] - 9I[0,6,4,2]$$
$$+ 3I[0,6,4,3] + 3I[0,6,5,0] - 6I[0,6,5,1] + 3I[0,6,5,2] - I[0,6,6,0] + I[0,6,6,1] + 6I[1,5,3,1] - 18I[1,5,3,2]$$
$$+ 18I[1,5,3,3] - 6I[1,5,3,4] - 18I[1,5,4,1] + 36I[1,5,4,2] - 18I[1,5,4,3] + 18I[1,5,5,1]$$
$$- 18I[1,5,5,2] - 6I[1,5,6,1])) + 20\pi^2 f_{3\gamma} P_3 m_s (4(32e_s + 41e_u)I[0,3,4,0] + 3(9e_s + 20e_u)I[0,3,5,0])I_2[\mathcal{V}]$$
$$+ 3m_0^2(e_s + 14e_u)I_4[\tilde{S}]I[0,5,4,0]\}, \tag{A3}$$

and

$$\Delta_4^{\mathrm{QCD}} = \frac{1}{47185920\pi^5} \{4e_c m_c(-480\pi^2 P_1 (P_2 - P_3)(I[0,2,1,0] - 2I[0,2,1,1] + I[0,2,1,2] - 2I[0,2,2,0]$$
$$+ 2I[0,2,2,1] + I[0,2,3,0] - 2(I[1,1,1,0] - 2I[1,1,1,1] + I[1,1,1,2] - 2I[1,1,2,0] + 2I[1,1,2,1]$$
$$+ I[1,1,3,0])) + 20m_s P_1 (7I[0,3,1,0] - 20I[0,3,1,1] + 19I[0,3,1,2] - 6I[0,3,1,3] - 14I[0,3,2,0]$$
$$+ 26I[0,3,2,1] - 12I[0,3,2,2] + 7I[0,3,3,0] - 6I[0,3,3,1] + 18(I[1,2,1,1] - 2I[1,2,1,2] + I[1,2,1,3]$$
$$- 2I[1,2,2,1] + 2I[1,2,2,2] + I[1,2,3,1])) + 360\pi^2(P_2 - P_3)(-3I[0,4,1,0] + 9I[0,4,1,1] - 9I[0,4,1,2]$$
$$+ 3I[0,4,1,3] + 10I[0,4,2,0] - 21I[0,4,2,1] + 12I[0,4,2,2] - I[0,4,2,3] - 11I[0,4,3,0] + 13I[0,4,3,1]$$
$$- 2I[0,4,3,2] + 4I[0,4,4,0] - I[0,4,4,1] + m_0^2(I[0,3,2,0] - 2I[0,3,2,1] + I[0,3,2,2] - 2I[0,3,3,0]$$
$$+ 2I[0,3,3,1] + I[0,3,4,0] + 6I[1,2,2,1] - 3I[1,2,2,2] - 6I[1,2,3,1]) + 4(I[1,3,2,1] - 2I[1,3,2,2]$$
$$+ I[1,3,2,3] - 2I[1,3,3,1] + 2I[1,3,3,2] + I[1,3,4,1])) - 27m_s(6I[0,5,1,1] - 18I[0,5,1,2] + 18I[0,5,1,3]$$
$$- 6I[0,5,1,4] - 20I[0,5,2,1] + 41I[0,5,2,2] - 22I[0,5,2,3] + I[0,5,2,4] + 22I[0,5,3,1] - 24I[0,5,3,2]$$
$$+ 2I[0,5,3,3] - 8I[0,5,4,1] + I[0,5,4,2] - 5(I[1,4,2,2] - 2I[1,4,2,3] + I[1,4,2,4] - 2I[1,4,3,2]$$
$$+ 2I[1,4,3,3] + I[1,4,4,2])))$$
$$- e_s(-15P_1(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2]$$
$$+ 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 4(3I[1,3,2,1] - 3I[1,3,2,2] + I[1,3,2,3] + 3(-2I[1,3,3,1]$$
$$+ I[1,3,3,2] + I[1,3,4,1]))) + 6480\pi^2 m_c P_2(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0]$$
$$+ 6I[0,4,3,1] - 3I[0,4,3,2] + 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 4(3I[1,3,2,1] - 3I[1,3,2,2]$$
$$+ I[1,3,2,3] + 3(-2I[1,3,3,1] + I[1,3,3,2] + I[1,3,4,1]))) + 8m_c^2(20P_1(I[0,3,1,0] - 3I[0,3,1,1]$$
$$+ 3I[0,3,1,2] - I[0,3,1,3] - 2I[0,3,2,0] + 4I[0,3,2,1] - 2I[0,3,2,2] + I[0,3,3,0] - I[0,3,3,1] + 3(I[1,2,1,1]$$
$$- 2I[1,2,1,2] + I[1,2,1,3] - 2I[1,2,2,1] + 2I[1,2,2,2] + I[1,2,3,1])) + 27(2I[0,5,2,1] - 5I[0,5,2,2]$$
$$+ 4I[0,5,2,3] - I[0,5,2,4] - 4I[0,5,3,1] + 6I[0,5,3,2] - 2I[0,5,3,3] + 2I[0,5,4,1] - I[0,5,4,2]$$
$$+ 5(I[1,4,2,2] - 2I[1,4,2,3] + I[1,4,2,4] - 2I[1,4,3,2] + 2I[1,4,3,3] + I[1,4,4,2]))) + 243(I[0,6,3,0]$$
$$- 4I[0,6,3,1] + 6I[0,6,3,2] - 4I[0,6,3,3] + I[0,6,3,4] - 3I[0,6,4,0] + 9I[0,6,4,1] - 9I[0,6,4,2]$$
$$+ 3I[0,6,4,3] + 3I[0,6,5,0] - 6I[0,6,5,1] + 3I[0,6,5,2] - I[0,6,6,0] + I[0,6,6,1] + 6(I[1,5,3,1]$$
$$- 3I[1,5,3,2] + 3I[1,5,3,3] - I[1,5,3,4] - 3(I[1,5,4,1] - 2I[1,5,4,2] + I[1,5,4,3] - I[1,5,5,1] + I[1,5,5,2])$$
$$- I[1,5,6,1])))$$
$$+ e_u(-15P_1(I[0,4,2,0] - 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2]$$
$$+ 3I[0,4,4,0] - 3I[0,4,4,1] - I[0,4,5,0] + 4(3I[1,3,2,1] - 3I[1,3,2,2] + I[1,3,2,3] + 3(-2I[1,3,3,1]$$
$$+ I[1,3,3,2] + I[1,3,4,1]))) + 8m_c^2(-720\pi^2 m_s P_3(I[0,3,2,0] - 2I[0,3,2,1] + I[0,3,2,2] - 2I[0,3,3,0]$$
$$+ 2I[0,3,3,1] + I[0,3,4,0] + 6I[1,2,2,1] - 3I[1,2,2,2] - 6I[1,2,3,1]) + 20P_1(I[0,3,1,0] - 3I[0,3,1,1]$$
$$+ 3I[0,3,1,2] - I[0,3,1,3] - 2I[0,3,2,0] + 4I[0,3,2,1] - 2I[0,3,2,2] + I[0,3,3,0] - I[0,3,3,1] + 3(I[1,2,1,1]$$

$$-2I[1,2,1,2] + I[1,2,1,3] - 2I[1,2,2,1] + 2I[1,2,2,2] + I[1,2,3,1])) + 27(2I[0,5,2,1] - 5I[0,5,2,2]$$

$$+ 4I[0,5,2,3] - I[0,5,2,4] - 4I[0,5,3,1] + 6I[0,5,3,2] - 2I[0,5,3,3] + 2I[0,5,4,1] - I[0,5,4,2]$$

$$+ 5(I[1,4,2,2] - 2I[1,4,2,3] + I[1,4,2,4] - 2I[1,4,3,2] + 2I[1,4,3,3] + I[1,4,4,2])))) + 324m_c(20\pi^2 P_3(I[0,4,2,0]$$

$$- 3I[0,4,2,1] + 3I[0,4,2,2] - I[0,4,2,3] - 3I[0,4,3,0] + 6I[0,4,3,1] - 3I[0,4,3,2] + 3I[0,4,4,0] - 3I[0,4,4,1]$$

$$- I[0,4,5,0] + 4(3I[1,3,2,1] - 3I[1,3,2,2] + I[1,3,2,3] + 3(-2I[1,3,3,1] + I[1,3,3,2] + I[1,3,4,1])))$$

$$+ 3m_s(I[0,5,2,0] - 4I[0,5,2,1] + 6I[0,5,2,2] - 4I[0,5,2,3] + I[0,5,2,4] - 3I[0,5,3,0] + 9I[0,5,3,1] - 9I[0,5,3,2]$$

$$+ 3I[0,5,3,3] + 3I[0,5,4,0] - 6I[0,5,4,1] + 3I[0,5,4,2] - I[0,5,5,0] + I[0,5,5,1] + 5(I[1,4,2,1] - 3I[1,4,2,2]$$

$$+ 3I[1,4,2,3] - I[1,4,2,4] - 3(I[1,4,3,1] - 2I[1,4,3,2] + I[1,4,3,3] - I[1,4,4,1] + I[1,4,4,2]) - I[1,4,5,1])))$$

$$+ 243(I[0,6,3,0] - 4I[0,6,3,1] + 6I[0,6,3,2] - 4I[0,6,3,3] + I[0,6,3,4] - 3I[0,6,4,0] + 9I[0,6,4,1] - 9I[0,6,4,2]$$

$$+ 3I[0,6,4,3] + 3I[0,6,5,0] - 6I[0,6,5,1] + 3I[0,6,5,2] - I[0,6,6,0] + I[0,6,6,1] + 6(I[1,5,3,1] - 3I[1,5,3,2]$$

$$+ 3I[1,5,3,3] - I[1,5,3,4] - 3(I[1,5,4,1] - 2I[1,5,4,2] + I[1,5,4,3] - I[1,5,5,1] + I[1,5,5,2]) - I[1,5,6,1])))$$

$$+ m_0^2(-120(e_s + 10e_u)f_{3\gamma}\pi^2 I_2[\mathcal{V}]I[0,4,4,0] + (3e_s + 14e_u)I_4[\mathcal{S}]I[0,5,4,0] + 2e_s I_4[\mathcal{T}_1]I[0,5,4,0])\}, \tag{A4}$$

where $P_1 = \langle g_s^2 G^2 \rangle$, $P_2 = \langle \bar{q}q \rangle$ and $P_3 = \langle \bar{s}s \rangle$ are gluon, u/d and s quark condensates, respectively. It should be noted that in the above terms, for the sake of simplicity, we only present expressions that make significant contributions to the numerical values of the magnetic moments and do not give much higher dimensional contributions, although they are taken into account in numerical computations.

The functions $I[n,m,l,k]$, $I_1[\mathcal{A}]$, $I_2[\mathcal{A}]$, $I_3[\mathcal{A}]$, $I_4[\mathcal{A}]$, $I_5[\mathcal{A}]$ and $I_6[\mathcal{A}]$ are defined as

$$I[n,m,l,k] = \int_{4m_c^2}^{s_0} ds \int_0^1 dt \int_0^1 dw\, e^{-s/M^2} s^n (s - 4m_c^2)^m t^l w^k,$$

$$I_1[\mathcal{A}] = \int D_{\alpha_i} \int_0^1 dv \mathcal{A}(\alpha_{\bar{q}}, \alpha_q, \alpha_g) \delta'(\alpha_q + \bar{v}\alpha_g - u_0),$$

$$I_2[\mathcal{A}] = \int D_{\alpha_i} \int_0^1 dv \mathcal{A}(\alpha_{\bar{q}}, \alpha_q, \alpha_g) \delta'(\alpha_{\bar{q}} + v\alpha_g - u_0),$$

$$I_3[\mathcal{A}] = \int D_{\alpha_i} \int_0^1 dv \mathcal{A}(\alpha_{\bar{q}}, \alpha_q, \alpha_g) \delta(\alpha_q + \bar{v}\alpha_g - u_0),$$

$$I_4[\mathcal{A}] = \int D_{\alpha_i} \int_0^1 dv \mathcal{A}(\alpha_{\bar{q}}, \alpha_q, \alpha_g) \delta(\alpha_{\bar{q}} + v\alpha_g - u_0),$$

$$I_5[\mathcal{A}] = \int_0^1 du A(u) \delta'(u - u_0),$$

$$I_6[\mathcal{A}] = \int_0^1 du A(u),$$

where $\mathcal{A}$ represents the corresponding photon DAs.

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