



Magnetic dipole moments of the spin- $\frac{3}{2}$ doubly heavy baryons

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Abstract The magnetic dipole moments of the spin- $\frac{3}{2}$ doubly charmed, bottom and charmed-bottom baryons are obtained by means of the light-cone QCD sum rule. The magnetic dipole moments of these baryons encode essential knowledge of their inner structure and shape deformations. The numerical results are given by $\mu_{\Xi_{cc}^{*++}} = 2.94 \pm 0.95$, $\mu_{\Xi_{cc}^{*+}} = -0.67 \pm 0.11$, $\mu_{\Omega_{cc}^{*+}} = -0.52 \pm 0.07$, $\mu_{\Xi_{bb}^{*0}} = 2.30 \pm 0.55$, $\mu_{\Xi_{bb}^{*-}} = -1.39 \pm 0.32$, $\mu_{\Omega_{bb}^{*-}} = -1.56 \pm 0.33$, $\mu_{\Xi_{bc}^{*+}} = 2.63 \pm 0.82$, $\mu_{\Xi_{bc}^{*0}} = -0.96 \pm 0.32$ and $\mu_{\Omega_{bc}^{*+}} = -1.11 \pm 0.33$, respectively.

1 Motivation

The doubly heavy baryons (DHBs) presumably contain two heavy quark and one light quark. One of them was firstly announced by the SELEX Collaboration in the decay mode $\Xi_{cc}^+ \rightarrow \Lambda_c^+ K^- \pi^+$ with the mass $M_{\Xi_{cc}^+} = 3519 \pm 1$ MeV [1]. However, neither Belle [2], nor FOCUS [3], nor BABAR [4] could confirm the DHBs in $e^- e^+$ annihilations. It is worth pointing out that the analysis of the SELEX experiment with other experimental groups is achieved through different production mechanisms. Therefore, the results of the SELEX Collaboration cannot be ruled out.

In 2017, LHCb Collaboration observed another doubly heavy baryon Ξ_{cc}^{++} in the mass spectrum of $\Lambda_c^+ K^- \pi^+ \pi^+$ with the mass $M_{\Xi_{cc}^{++}} = 3621.40 \pm 0.72 \pm 0.27 \pm 0.14$ MeV [5]. Recently, the LHCb Collaboration reconstructed their analysis via decay modes $\Xi_{cc}^{++} \rightarrow \Lambda_c^+ K^- \pi^+ \pi^+$ and $\Xi_{cc}^{++} \rightarrow \Xi_c^+ \pi^+$ and they reported their mass value as $M_{\Xi_{cc}^{++}} = 3621.55 \pm 0.23 \pm 0.30$ MeV [6]. The investigation for the DHBs may provide with valuable knowledge for comprehension of the nonperturbative QCD effects. One of the several point of views which makes the physics of DHBs charming is that the binding of two charm quarks and a light quark provides a unique perspective for dynamics of confine-

ment. The research of the properties of DHBs is one of the active and interesting branches of particle physics. Therefore, the weak [7–13], strong [14, 15] and radiative decays [16–19], the magnetic dipole moments (MDMs) [20–39] and masses [7, 14, 15, 18, 20–26, 40–77] of the DHBs have been examined broadly in the literature with the help of the quark models, potential models, lattice QCD, the Feynman–Hellmann theorem, an extended chromomagnetic model, chiral perturbation theory, heavy quark effective theory, QCD sum rules, perturbative QCD, the Faddeev approach, SU(3) flavor symmetry, a nonperturbative string approach, a local diquark approach, light-cone QCD sum rules, a light front approach and an extended on-mass-shell renormalization scheme.

Electromagnetic properties correspond to the major and meaningful parameters of the DHBs. As the electromagnetic properties symbolize necessary viewpoints of the intrinsic properties of hadrons, it is quite significant to analyze the baryon electromagnetic form factors, particularly the MDMs. The magnitude and sign of the MDM ensure crucial data on size, structure and shape deformations of baryons. Apparently, determining the MDM is an important step in our comprehension of the baryon features with regard to quark–gluon degrees of freedom. In this study, we are going to concentrate on the DHBs (from now on we will represent these particles as B_{QQ}^*) with spin–parity $J^P = \frac{3}{2}^+$, and calculate their MDMs with the help of the light-cone QCD sum rule (LCSR) approach, which is one of the powerful nonperturbative methods in hadron physics making it possible for us to calculate properties of the particles and processes. In LCSR, the hadronic properties are expressed with regard to the vacuum condensates and the light-cone distribution amplitudes of the on-shell particles [78–80]. Since the MDMs are quantities described in terms of the properties of the vacuum and distribution amplitudes of the particles, any uncertainties in these parameters are reflected by the uncertainties of the predictions of the MDMs. The first extraction of the MDMs of

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the spin- $\frac{3}{2}$ doubly heavy baryons were obtained by Lichtenberg [32] by means of the naive quark model. Many scientists have used different theoretical models to compute the spin- $\frac{3}{2}$ doubly heavy baryon MDMs [7, 26, 31, 34, 37, 38, 70, 81, 82]. In Ref. [7], the static properties and semileptonic decays of the DHBs with the help of nonrelativistic quark model have been calculated. To examine the dependence of the results on inter-quark interaction one uses various quark potential models that contain hyperfine terms coming from one gluon exchange and a Coulomb term as well. In Refs. [70, 81], the masses of the ground, orbital and radial states of the DHBs are evaluated in the framework of the hypercentral constituent quark model with Coulomb plus linear potential. The MDMs of the ground state DHBs are also extracted. In Ref. [37], the MDMs of the spin- $\frac{3}{2}$ doubly charmed baryons are investigated up to NNLO by means of the heavy baryon chiral perturbation theory. As a by-product, one obtained the numerical values of the doubly bottom and charmed-bottom baryons up to NLO. In Refs. [38, 82], the MDMs of singly and DHBs with spin- $\frac{3}{2}$ and spin- $\frac{1}{2}$ in the framework of effective quark mass and screened charge of quark are calculated. In Ref. [31], the MDMs of the DHBs are investigated in the MIT bag model with center of mass motion corrections. In Ref. [34], the decay widths, MDMs and M1 transitions of all ground state heavy baryons are evaluated by means of the extended MIT bag model. In Ref. [26], the extended relativistic harmonic confinement model is utilized to acquire masses and MDMs of the heavy flavored ground state baryons.

The plan of the manuscript is as follows. In Sect. 2, the details of the MDMs computations for the DHBs with spin- $\frac{3}{2}$ are presented. Numerical analysis of the LCSR for the MDMs are given in Sect. 3. Section 4 is reserved for discussion and concluding remarks.

2 Formalism

To obtain the MDM of the DHBs by using the LCSR approach, we begin with the subsequent correlation function,

$$\Pi_{\mu\nu\alpha}(p, q) = i^2 \int d^4x \int d^4y e^{ip \cdot x + iq \cdot y} \times \left\langle 0 \left| \mathcal{T} \left\{ J_\mu^{B^*_{QQ}}(x) J_\nu^{B^*_{QQ}}(y) J_\alpha^{B^*_{QQ}}(0) \right\} \right| 0 \right\rangle. \quad (1)$$

Here, $J_{\mu(v)}$ is the interpolating current of the B^*_{QQ} baryons and the electromagnetic current J_α is given by

$$J_\alpha = \sum_{q=u,d,s,c,b} e_q \bar{q} \gamma_\alpha q, \quad (2)$$

where e_q is the electric charge of the corresponding quark.

From a technical point of view, Eq. (1) can be reconsidered in a more appropriate form with the help of an external background electromagnetic (EBGEM) field,

$$\Pi_{\mu\nu}(p, q) = i \int d^4x e^{ip \cdot x} \left\langle 0 \left| \mathcal{T} \left\{ J_\mu^{B^*_{QQ}}(x) J_\nu^{B^*_{QQ}}(0) \right\} \right| 0 \right\rangle_F, \quad (3)$$

where F is the EBGEM field and $F_{\alpha\beta} = i(\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha)$ with ε_β and q_α being the polarization and four-momentum of the EBGEM field, respectively. Since the EBGEM field can be made arbitrarily small, the correlation function in Eq. (3) can be acquired by expanding in powers of the EBGEM field,

$$\Pi_{\mu\nu}(p, q) = \Pi_{\mu\nu}^{(0)}(p, q) + \Pi_{\mu\nu}^{(1)}(p, q) + \dots, \quad (4)$$

and keeping only terms $\Pi_{\mu\nu}^{(1)}(p, q)$, which corresponds to the single photon emission [83]. The primary advantage of using the EBGEM field approach relies on the fact that it separates the soft and hard photon emissions in an explicitly gauge invariant way [83]. The $\Pi_{\mu\nu}^{(0)}(p, q)$ is the correlation function while the EBGEM field is lacking, and this leads to the two point sum rules of the hadrons, which is not relevant for our case.

After these general comments, we can now move to deriving the LCSR for the MDM of the DHBs. The correlation function given in Eq. (3) can be calculated with regard to hadronic properties; this is known as hadronic representation. In addition to this results can be obtained with regard to the quark–gluon properties in the deep Euclidean region, known as QCD representation. By matching the results of these representations using the dispersion relation and the quark–hadron duality ansatz, one can acquire the corresponding sum rules.

We begin to evaluate the correlation function with respect to hadronic degrees of freedom comprising the physical properties of the particles under investigation. For that purpose, we embed a complete set of B^*_{QQ} baryons into the correlation function. So, we get

$$\Pi_{\mu\nu}^{Had}(p, q) = \frac{\langle 0 | J_\mu^{B^*_{QQ}} | B^*_{QQ}(p) \rangle}{[p^2 - m_{B^*_{QQ}}^2]} \langle B^*_{QQ}(p) | B^*_{QQ}(p+q) \rangle_F \frac{\langle B^*_{QQ}(p+q) | \bar{J}_\nu^{B^*_{QQ}} | 0 \rangle}{[(p+q)^2 - m_{B^*_{QQ}}^2]} + \dots, \quad (5)$$

where the dots stand for contributions of higher states and the continuum. The matrix elements in Eq. (5) are defined as [84, 85]

$$\langle 0 | J_\mu(0) | B^*_{QQ}(p, s) \rangle = \lambda_{B^*_{QQ}} u_\mu(p, s), \quad (6)$$

$$\begin{aligned}
 & \langle B_{QQ}^*(p) | B_{QQ}^*(p+q) \rangle_F \\
 &= -e \bar{u}_\mu(p) \left\{ F_1(q^2) g_{\mu\nu} \not{q} \right. \\
 &\quad - \frac{1}{2m_{B_{QQ}^*}} \left[F_2(q^2) g_{\mu\nu} + F_4(q^2) \right. \\
 &\quad \times \left. \frac{q_\mu q_\nu}{(2m_{B_{QQ}^*})^2} \right] \not{q} + \frac{F_3(q^2)}{(2m_{B_{QQ}^*})^2} \\
 &\quad \left. \times q_\mu q_\nu \not{q} \right\} u_\nu(p+q), \tag{7}
 \end{aligned}$$

where $\lambda_{B_{QQ}^*}$ is the residue of the B_{QQ}^* baryon and $u_\mu(p, s)$ is the Rarita–Schwinger spinor. Summation over spins of the B_{QQ}^* baryon is carried out as follows:

$$\begin{aligned}
 \sum_s u_\mu(p, s) \bar{u}_\nu(p, s) = & -(\not{p} + m_{B_{QQ}^*}) \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu \right. \\
 & \left. - \frac{2 p_\mu p_\nu}{3 m_{B_{QQ}^*}^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3 m_{B_{QQ}^*}} \right]. \tag{8}
 \end{aligned}$$

Substituting Eqs. (5)–(8) into Eq. (3) for hadronic side we obtain

$$\begin{aligned}
 \Pi_{\mu\nu}^{Had}(p, q) = & - \frac{\lambda_{B_{QQ}^*}^2 (\not{p} + m_{B_{QQ}^*})}{\left[(p+q)^2 - m_{B_{QQ}^*}^2 \right] \left[p^2 - m_{B_{QQ}^*}^2 \right]} \\
 & \left[g_{\mu\nu} - \frac{1}{3} \gamma_\mu \gamma_\nu - \frac{2 p_\mu p_\nu}{3 m_{B_{QQ}^*}^2} + \frac{p_\mu \gamma_\nu - p_\nu \gamma_\mu}{3 m_{B_{QQ}^*}} \right] \\
 & \times \left\{ F_1(q^2) g_{\mu\nu} \not{q} - \frac{1}{2m_{B_{QQ}^*}} \left[F_2(q^2) g_{\mu\nu} \right. \right. \\
 & \left. \left. + F_4(q^2) \frac{q_\mu q_\nu}{(2m_{B_{QQ}^*})^2} \right] \not{q} + \frac{F_3(q^2)}{(2m_{B_{QQ}^*})^2} q_\mu q_\nu \not{q} \right\}. \tag{9}
 \end{aligned}$$

As a principle, we can achieve the last form of the hadronic representation of the correlator using Eq. (9), however, we are faced with two problems. One of them is related to the fact that not all Lorentz structures appearing in Eq. (9) are independent. The second problem is that the correlator can also get contributions from spin-1/2 baryons, which should be removed. To eliminate the spin-1/2 contributions and acquire just independent structures in the correlator, we perform the ordering for Dirac matrices as follows: $\gamma_\mu \not{p} \not{q} \gamma_\nu$ and remove terms with γ_μ at the beginning, γ_ν at the end and all those proportional to p_μ and p_ν [86]. As a result, for the hadronic side we get

$$\begin{aligned}
 \Pi_{\mu\nu}^{Had}(p, q) = & - \frac{\lambda_{B_{QQ}^*}^2}{\left[(p+q)^2 - m_{B_{QQ}^*}^2 \right] \left[p^2 - m_{B_{QQ}^*}^2 \right]} \\
 & \times \left[-g_{\mu\nu} \not{p} \not{q} F_1(q^2) + m_{B_{QQ}^*} g_{\mu\nu} \not{q} F_2(q^2) \right. \\
 & \left. + \text{other independent structures} \right]. \tag{10}
 \end{aligned}$$

The MDM form factor, $G_M(q^2)$, is defined with respect to the form factors $F_i(q^2)$ in the following manner [84,85]:

$$\begin{aligned}
 G_M(q^2) = & \left[F_1(q^2) + F_2(q^2) \right] \left(1 + \frac{4}{5} \tau \right) - \frac{2}{5} \left[F_3(q^2) \right. \\
 & \left. + F_4(q^2) \right] \tau (1 + \tau), \tag{11}
 \end{aligned}$$

where $\tau = -\frac{q^2}{4m_{B_{QQ}^*}^2}$. At $q^2 = 0$, the magnetic dipole form factors are obtained with respect to the functions $F_i(0)$:

$$G_M(0) = F_1(0) + F_2(0). \tag{12}$$

The MDM ($\mu_{B_{QQ}^*}$) is defined in the following way:

$$\mu_{B_{QQ}^*} = \frac{e}{2m_{B_{QQ}^*}} G_M(0). \tag{13}$$

In this study we achieve QCD sum rules for the form factors $F_i(q^2)$ at first, after that in numerical calculations we will make use of the above equations to obtain the values of the MDMs using the QCD sum rules for the form factors. The final form of the hadronic representation with respect to the chosen structures in momentum space is

$$\Pi_{\mu\nu}^{Had}(p, q) = \Pi_1^{Had} g_{\mu\nu} \not{p} \not{q} + \Pi_2^{Had} g_{\mu\nu} \not{q} + \dots, \tag{14}$$

where Π_i^{Had} are functions of the form factors $F_i(q^2)$ and other hadronic parameters; and ... represents other independent structures.

To obtain the expression of the correlation function with respect to the quark–gluon parameters, the explicit form for the interpolating current of the B_{QQ}^* baryons needs to be chosen. In this work, we consider the B_{QQ}^* baryons with the quantum numbers $J^P = \frac{3}{2}^+$. The interpolating current is given as [64]

$$\begin{aligned}
 J_\mu^{B_{QQ}^*}(x) = & \frac{1}{\sqrt{3}} \epsilon^{abc} \left\{ (q^{aT} C \gamma_\mu Q^b) Q'^c + (q^{aT} C \gamma_\mu Q'^b) Q^c \right. \\
 & \left. + (Q^{aT} C \gamma_\mu Q'^b) q^c \right\}, \tag{15}
 \end{aligned}$$

where q is the light; and Q and Q' are the two heavy quarks, respectively. We give the quark content of the spin-3/2 DHBs in Table 1.

Table 1 The quark content of the spin-3/2 DHBs

Baryon	q	Q	Q'
Ξ_{QQ}^*	u or d	b or c	b or c
$\Xi_{QQ'}^*$	u or d	b	c
Ω_{QQ}^*	s	b or c	b or c
$\Omega_{QQ'}^*$	s	b	c

After contracting pairs of quark fields and using Wick's theorem, the correlation function becomes

$$\begin{aligned}
 \Pi_{\mu\nu}^{QCD}(p) &= -\frac{i}{3} \varepsilon^{abc} \varepsilon^{a'b'c'} \int [d^4x e^{ip \cdot x} \langle 0 | \left\{ S_{Q'}^{cc'} \text{Tr} \left[S_Q^{ba'} \gamma_\nu \tilde{S}_q^{ab'} \gamma_\mu \right] \right. \\
 &+ S_q^{cc'} \text{Tr} \left[S_{Q'}^{ba'} \gamma_\nu \tilde{S}_Q^{ab'} \gamma_\mu \right] \\
 &+ S_Q^{cc'} \text{Tr} \left[S_q^{ba'} \gamma_\nu \tilde{S}_{Q'}^{ab'} \gamma_\mu \right] + S_Q^{cb'} \gamma_\nu \tilde{S}_{Q'}^{aa'} \gamma_\mu S_q^{bc'} \\
 &+ S_Q^{ca'} \gamma_\nu \tilde{S}_q^{bb'} \gamma_\mu S_{Q'}^{ac'} \\
 &+ S_{Q'}^{ca'} \gamma_\nu \tilde{S}_Q^{bb'} \gamma_\mu S_q^{ac'} + S_{Q'}^{cb'} \gamma_\nu \tilde{S}_q^{aa'} \gamma_\mu S_Q^{bc'} \\
 &+ S_q^{ca'} \gamma_\nu \tilde{S}_{Q'}^{bb'} \gamma_\mu S_Q^{ac'} \\
 &\left. + S_q^{cb'} \gamma_\nu \tilde{S}_Q^{aa'} \gamma_\mu S_{Q'}^{bc'} \right\} | 0 \rangle_F, \tag{16}
 \end{aligned}$$

where $\tilde{S}_{Q(q)}^{ij}(x) = CS_{Q(q)}^{ijT}(x)C$ and, $S_q^{ij}(x)$ and $S_Q^{ij}(x)$ are the light and heavy quark propagators, respectively. The light and heavy quark propagators are given as [87,88]

$$\begin{aligned}
 S_q(x) &= S_q^{free} - \frac{\bar{q}q}{12} \left(1 - i \frac{m_q \not{x}}{4} \right) - \frac{\bar{q}\sigma \cdot Gq}{192} x^2 \left(1 - i \frac{m_q \not{x}}{6} \right) \\
 &\quad - \frac{igs}{32\pi^2 x^2} G^{\mu\nu}(x) [k\sigma_{\mu\nu} + \sigma_{\mu\nu}k], \\
 S_Q(x) &= S_Q^{free} - \frac{gs m_Q}{16\pi^2} \int_0^1 dv G^{\mu\nu}(vx) [(\sigma_{\mu\nu} \not{x} + \not{x} \sigma_{\mu\nu}) \\
 &\quad \times \frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + 2\sigma^{\mu\nu} K_0(m_Q \sqrt{-x^2})], \tag{17}
 \end{aligned}$$

where

$$\begin{aligned}
 S_q^{free} &= \frac{1}{2\pi^2 x^2} \left(i \frac{\not{x}}{x^2} - \frac{m_q}{2} \right), \\
 S_Q^{free} &= \frac{m_Q^2}{4\pi^2} \left[\frac{K_1(m_Q \sqrt{-x^2})}{\sqrt{-x^2}} + i \frac{\not{x} K_2(m_Q \sqrt{-x^2})}{(\sqrt{-x^2})^2} \right], \tag{18}
 \end{aligned}$$

with $G^{\mu\nu}$ is the gluon field strength tensor, K_i are second kind of the Bessel functions, m_q and m_Q are the light and heavy quark mass, respectively.

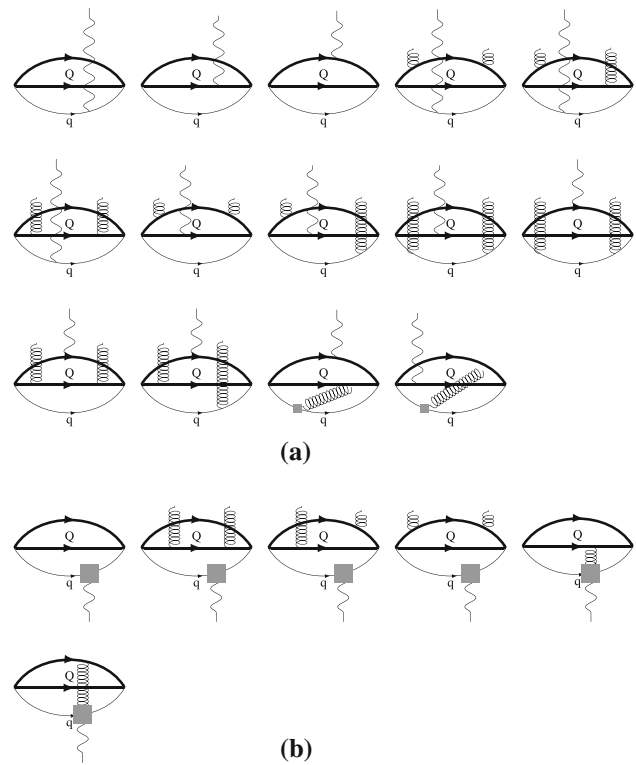


Fig. 1 Feynman diagrams for the MDMs of the spin-3/2 DHBs. The thick, thin, wavy and curly lines denote the heavy quark, light quark, photon and gluon propagators, respectively. Diagrams (a) correspond to the perturbative photon vertex and diagrams (b) denote the contributions coming from the distribution amplitudes of the photon

The correlator in Eq. (16) contains various contributions: the photon can be emitted both perturbatively or nonperturbatively (see Fig. 1). When the photon is emitted perturbatively, one of the propagators in Eq. (16) is substituted by

$$S_q^{free}(x) \rightarrow \int d^4y S_q^{free}(x-y) A(y) S_q^{free}(y), \tag{19}$$

and the surviving two propagators are substituted with the full quark propagators including the free (perturbative) part as well as the interacting parts (with gluon or QCD vacuum) as nonperturbative contributions. The total perturbative photon emission is achieved by carrying out the substitution mentioned above for the perturbatively interacting quark propagator with the photon and employing the substitution of the surviving propagators by their free parts.

In the case of nonperturbative photon emission, the light quark propagator in Eq. (16) is substituted by

$$S_{\alpha\beta}^{ab} \rightarrow -\frac{1}{4} (\bar{q}^a \Gamma_i q^b) (\Gamma_i)_{\alpha\beta}, \tag{20}$$

where Γ_i represent the full set of Dirac matrices. In this approach, two surviving quark propagators are taken as the full propagators comprising perturbative as well as non-

perturbative contributions. Once Eq. (20) is inserted into Eq. (16), there seem to occur matrix elements such as $\langle \gamma(q) | \bar{q}(x) \Gamma_i q(0) | 0 \rangle$ and $\langle \gamma(q) | \bar{q}(x) \Gamma_i G_{\alpha\beta} q(0) | 0 \rangle$, representing the nonperturbative contributions. Furthermore, nonlocal operators such as $\bar{q}q\bar{q}q$ and $\bar{q}G^2q$ are anticipated to appear. In this study, we take into account operators that include only one gluon field and three particle nonlocal operators and we disregard terms with four quarks $\bar{q}q\bar{q}q$, and two gluons $\bar{q}G^2q$. In order to calculate the nonperturbative contributions, we need the matrix elements of the nonlocal operators between the photon states and the vacuum and these matrix elements are described with respect to the photon distribution amplitudes with definite twists. Up to twist-4 the explicit expressions of the photon distribution amplitudes are given in [83]. Using these expressions for the propagators and distribution amplitudes for the photon, the correlation functions from the QCD side can be computed.

The QCD and hadronic representations of the correlation function are then matched using dispersion relation. The next step in deriving the sum rules for the MDMs of the spin- $\frac{3}{2}$ DHBs is applying double Borel transformations (\mathcal{B}) over the p^2 and $(p+q)^2$ on the two representations of the correlation function so as to suppress the contributions of higher states and continuum. As a result, we obtain

$$B\Pi_{\mu\nu}^{\text{Had}}(p, q) = B\Pi_{\mu\nu}^{\text{QCD}}(p, q), \tag{21}$$

which leads to

$$B\Pi_1^{\text{Had}} = B\Pi_1^{\text{QCD}}, \quad B\Pi_2^{\text{Had}} = B\Pi_2^{\text{QCD}}, \tag{22}$$

corresponding to the structures $g_{\mu\nu} \not{p} \not{q}$ and $g_{\mu\nu} \not{q}$. In this manner we extract the QCD sum rules for the form factors F_1 and F_2 . They are very lengthy functions; therefore we do not give their explicit expressions here. The interested reader can find details of the calculations such as Borel transformations and continuum subtraction in Refs. [89,90]

3 Numerical analysis

In this section, we achieve numerical computations for the spin- $\frac{3}{2}$ DHBs. We use $m_u = m_d = 0, m_s = 96_{-4}^{+8}$ MeV, $m_c = 1.67 \pm 0.07$ GeV, $m_b = 4.78 \pm 0.06$ GeV, $f_{3\gamma} = -0.0039$ GeV² [83], $\langle \bar{q}q \rangle = (-0.24 \pm 0.01)^3$ GeV³ [91], $m_0^2 = 0.8 \pm 0.1$ GeV², $\langle g_s^2 G^2 \rangle = 0.88$ GeV⁴ and $\chi = -2.85 \pm 0.5$ GeV⁻² [92]. The masses of the Ξ_{QQ}^* , $\Xi_{QQ'}^*$, Ω_{QQ}^* and $\Omega_{QQ'}^*$ baryons are borrowed from Ref. [64], in which the mass sum rules have been used to compute them. These masses are taken to have the following values: $M_{\Xi_{cc}^*} = 3.72 \pm 0.18$ GeV, $M_{\Omega_{cc}^*} = 3.78 \pm 0.16$ GeV, $M_{\Xi_{bc}^*} = 7.25 \pm 0.20$ GeV, $M_{\Omega_{bc}^*} = 7.30 \pm 0.20$ GeV, $M_{\Xi_{bb}^*} = 10.40 \pm 1.00$ GeV and $M_{\Omega_{bb}^*} = 10.50 \pm 0.20$ GeV. In order to specify the MDMs of DHBs, the values of the residues are needed. The residues of the DHBs are computed in Ref. [64]. These

residues are calculated to have the following values: $\lambda_{\Xi_{cc}^*} = 0.12 \pm 0.01$ GeV³, $\lambda_{\Omega_{cc}^*} = 0.14 \pm 0.02$ GeV³, $\lambda_{\Xi_{bc}^*} = 0.15 \pm 0.01$ GeV³, $\lambda_{\Omega_{bc}^*} = 0.18 \pm 0.02$ GeV³, $\lambda_{\Xi_{bb}^*} = 0.22 \pm 0.03$ GeV³ and $\lambda_{\Omega_{bb}^*} = 0.25 \pm 0.03$ GeV³. The parameters used in the photon distribution amplitudes are given in Ref. [83].

The QCD sum rule for the MDMs of the DHBs, besides the above-mentioned input parameters, include also two more extra parameters. These parameters are the continuum threshold s_0 and the Borel mass parameter M^2 . According to the QCD sum rules philosophy we need to find the working regions of these parameters, where the MDMs of the DHBs be insensitive to the variation of these parameters in their working regions. The s_0 is a not entirely optional parameter, it is preferred as the point at which the continuum and excited states begin to contribute to the calculations. To decide the working interval of the s_0 , we enforce the conditions of operator product expansion (OPE) convergence and pole dominance. Therefore, it is expected that s_0 varies in the interval $(M_{B_{QQ}^*} + 0.3)^2 \text{ GeV}^2 \leq s_0 \leq (M_{B_{QQ}^*} + 0.7)^2 \text{ GeV}^2$. From this point of view, we prefer the value of the s_0 within the interval $s_0 = (16-20) \text{ GeV}^2$ for the Ξ_{cc}^* , $s_0 = (58-62) \text{ GeV}^2$ for the Ξ_{bc}^* , $s_0 = (116-120) \text{ GeV}^2$ for Ξ_{bb}^* , $s_0 = (18-22) \text{ GeV}^2$ for the Ω_{cc}^* , $s_0 = (60-64) \text{ GeV}^2$ for the Ω_{bc}^* and $s_0 = (118-122) \text{ GeV}^2$ for the Ω_{bb}^* baryons. The working region for M^2 is found by requiring that the series of OPE in the QCD representation is convergent and the contribution of higher states and continuum is efficiently suppressed. In technical language, the upper limit on M^2 is found demanding the maximum pole contribution. The lower limit is obtained demanding that the contribution of the perturbative part exceeds the nonperturbative one and the series of the operator product expansion in the obtained sum rules converge. Our numerical calculations indicate that the two conditions are satisfied when M^2 changes in the regions: $4 \text{ GeV}^2 \leq M^2 \leq 6 \text{ GeV}^2$ for the Ξ_{cc}^* , $7 \text{ GeV}^2 \leq M^2 \leq 9 \text{ GeV}^2$ for the Ξ_{bc}^* , $10 \text{ GeV}^2 \leq M^2 \leq 14 \text{ GeV}^2$ for the Ξ_{bb}^* , $5 \text{ GeV}^2 \leq M^2 \leq 7 \text{ GeV}^2$ for the Ω_{cc}^* , $8 \text{ GeV}^2 \leq M^2 \leq 10 \text{ GeV}^2$ for the Ω_{bc}^* and $11 \text{ GeV}^2 \leq M^2 \leq 15 \text{ GeV}^2$ for the Ω_{bb}^* baryons. As an example in Fig. 2, we present the dependencies of the MDMs of doubly charmed baryons on M^2 at several fixed values of the s_0 . As is seen from the figure, although being not entirely insensitive, the MDMs exhibit an acceptable dependency on the extra parameters, s_0 and M^2 , which is reasonable in the error limits of the QCD sum rule formalism.

Our final results on the MDMs for the spin- $\frac{3}{2}$ DHBs are

$$\begin{aligned} \mu_{\Xi_{cc}^{*++}} &= 2.94 \pm 0.95, \\ \mu_{\Xi_{bc}^{*+}} &= 2.63 \pm 0.82, \\ \mu_{\Xi_{bb}^{*0}} &= 2.30 \pm 0.55, \end{aligned}$$

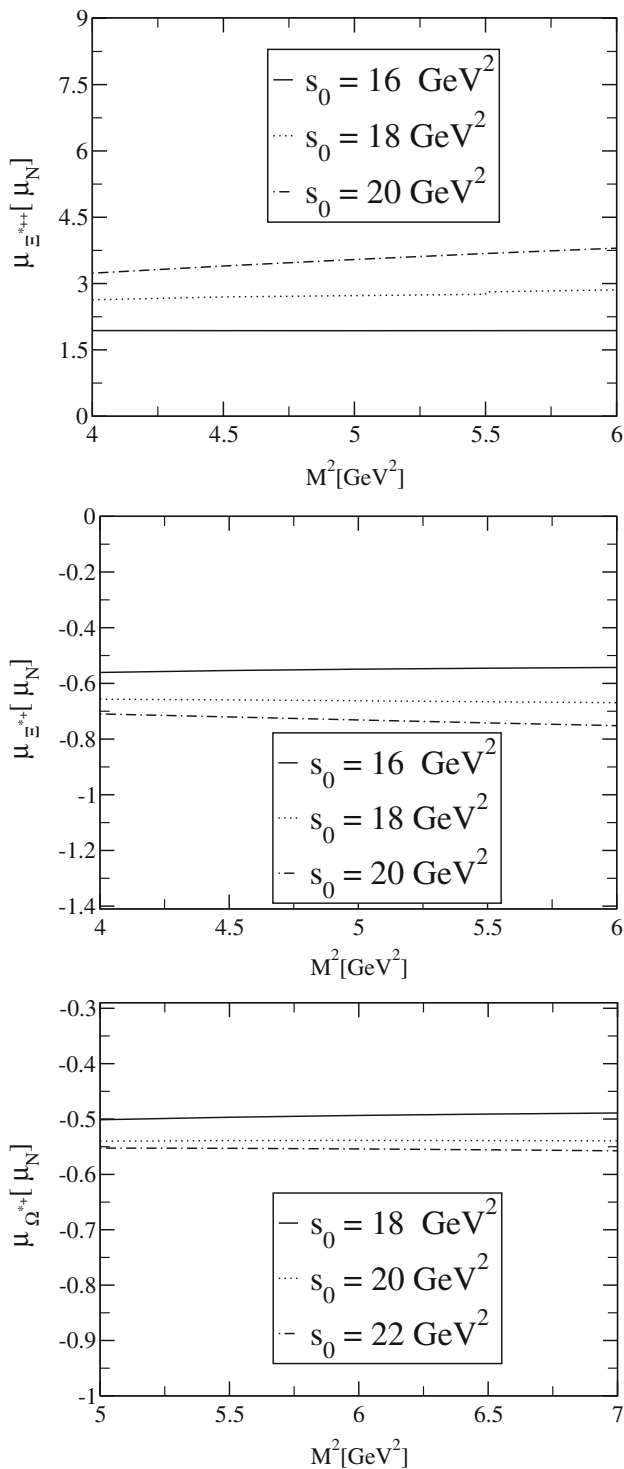


Fig. 2 The dependence of the MDMs for the spin- $\frac{3}{2}$ doubly charmed baryons on the M^2 at various fixed values of the s_0

$$\begin{aligned} \mu_{\Xi_{cc}^{*+}} &= -0.67 \pm 0.11, \\ \mu_{\Xi_{bc}^{*0}} &= -0.96 \pm 0.32, \\ \mu_{\Xi_{bb}^{*-}} &= -1.39 \pm 0.32, \\ \mu_{\Omega_{cc}^{*+}} &= -0.52 \pm 0.07, \end{aligned}$$

$$\begin{aligned} \mu_{\Omega_{bc}^{*+}} &= -1.11 \pm 0.33, \\ \mu_{\Omega_{bb}^{*-}} &= -1.56 \pm 0.33, \end{aligned} \tag{23}$$

where the quoted errors in the results are in connection with the uncertainties in the values of the input parameters and the photon distribution amplitudes, as well as the variations in the computations of the working windows M^2 and s_0 . We also need to emphasize that the main source of uncertainties is in the variations with respect to s_0 and the results weakly depend on the choices of the M^2 .

In Table 2, we present our numerical results for the MDMs and a comparison with various other models, such as the relativistic harmonic confinement model (RHM) [26], the MIT bag model [31,34], the nonrelativistic quark model (NRQM) [7], the hyper central constituent model (HCQM) [70,81], the effective mass (EMS) and screened charge scheme (SCS) [38,82] and heavy baryon chiral perturbation theory (HBChBT) [37]. From a comparison of our values with the predictions of other models we observe from this table that, for the Ξ_{cc}^{*++} baryon, practically all methods give similar predictions. For the Ξ_{cc}^{*+} and Ω_{cc}^{*+} baryons, there are large discrepancies among results not only as regards the magnitude but also as regards the sign.

The reason for such inconsistencies is relatively easy to understand. The sign of the MDM depends on what is stronger—two heavy quarks and one light quarks. In our analysis, the light quark overcome two heavy quarks and give the dominant contributions.

For the Ξ_{bb}^{*0} baryon, our estimation is consistent within the errors with Refs. [7,26,37] and unlike other approaches. For the Ξ_{bb}^{*-} baryon, nearly all models give similar predictions except the values of Refs. [31,34], which are small. For the Ω_{bb}^{*-} baryon, our estimation is consistent within the errors with Refs. [37,70,81] and different from other results. For the Ξ_{bc}^{*+} baryon, we see that within errors our predictions are in good agreement with the Refs. [7,26,37,38,82]. For the Ξ_{bc}^{*0} baryon, while the sign of the MDM is correctly determined, there is a large discrepancy among results. For the Ω_{bc}^{*0} baryon, nearly all methods give, moderately, similar approximations except the results of Ref. [37] and this work, which are quite large.

There is no experimental data for the MDMs of the DHBs. But one can define some useful splittings rules for the checking and comparing the results. In the heavy quark limit, the Ξ_{QQq}^* , $\Xi_{QQ'q}^*$, Ω_{QQq}^* and $\Omega_{QQ'q}^*$ baryons have the same MDMs. They have the same light degrees of freedom and the contribution of heavy quarks will vanish in this limit. If one keeps the heavy quark mass, the MDMs are not equal anymore. However, one can expect that they may satisfy the following relations as the $\Xi_{QQ'q}^*$ and $\Omega_{QQ'q}^*$ are intermediate states:

Table 2 MDMs of the spin- $\frac{3}{2}$ DHBs (in nuclear magnetons μ_N)

Approaches	Ξ_{cc}^{*++}	Ξ_{cc}^{*+}	Ω_{cc}^{*+}	Ξ_{bb}^{*0}	Ξ_{bb}^{*-}	Ω_{bb}^{*-}	Ξ_{bc}^{*+}	Ξ_{bc}^{*0}	Ω_{bc}^{*0}
NRQM [7]	2.67	-0.31	0.14	1.87	-1.11	-0.66	2.27	-0.71	-0.26
HCQM [70,81]	2.22	0.07	0.29	1.61	-1.74	-1.24	1.56	-0.38	-0.18
HBChBT-I [37]	3.51	-0.27	-0.64	2.83	-1.33	-1.54	3.22	-0.84	-1.09
HBChBT-II [37]	3.63	-0.37	-0.65	2.87	-1.38	-1.55	3.27	-0.89	-1.10
EMS [38,82]	2.41	-0.11	0.16	1.60	-0.98	-0.70	2.01	-0.55	-0.28
SCS [38,82]	2.52	0.04	0.21	1.50	-1.02	-0.80	2.02	-0.50	-0.30
MIT Bag model-I [31]	2.00	0.16	0.33	0.92	-0.65	-0.52	1.41	-0.25	-0.11
MIT Bag model-II [34]	2.35	-0.18	-0.05	1.40	-0.88	-0.70	1.88	-0.54	-0.33
RHM [26]	2.72	-0.23	0.16	2.30	-1.32	-0.86	2.68	-0.76	-0.32
This work	2.94	-0.67	-0.52	2.30	-1.39	-1.56	2.63	-0.96	-1.11

Table 3 Comparison of the first splitting

Approaches	$\Delta\mu_u$	$\Delta\mu_d$	$\Delta\mu_s$
NRQM [7]	0.00	0.00	0.00
HCQM [70,81]	0.71	-0.91	-0.59
HBChBT-I [37]	-0.10	0.08	0.00
HBChBT-II [37]	-0.04	0.03	0.00
EMS [38,82]	-0.01	0.01	0.02
SCS [38,82]	-0.02	0.02	0.01
MIT Bag model-I [31]	0.10	0.01	0.03
MIT Bag model-II [34]	-0.01	0.02	-0.09
RHM [26]	-0.34	-0.03	-0.06
This work	-0.02	-0.14	0.14

Table 4 Comparison of the second and third splittings

Approaches	$\Delta\mu_{u-d}^{cc}$	$\Delta\mu_{u-d}^{bb}$	$\Delta\mu_{u-d}^{bc}$	$\Delta\mu_{d-s}^{cc}$	$\Delta\mu_{d-s}^{bb}$	$\Delta\mu_{d-s}^{bc}$
NRQM [7]	2.98	2.98	2.98	-0.45	-0.45	-0.45
HCQM [70,81]	2.15	3.35	1.94	-0.22	-0.50	-0.20
HBChBT-I [37]	3.78	4.16	4.06	0.37	0.21	0.25
HBChBT-II [37]	4.00	4.25	4.16	0.28	0.17	0.21
EMS [38,82]	2.52	2.58	2.56	-0.27	-0.28	-0.27
SCS [38,82]	2.48	2.52	2.52	-0.17	-0.22	-0.20
MIT Bag model-I [31]	1.84	1.57	1.66	-0.17	-0.13	-0.14
MIT Bag model-II [34]	2.53	2.28	2.42	-0.13	-0.18	-0.21
RHM [26]	2.95	3.62	3.44	-0.39	-0.46	-0.44
This work	3.61	3.69	3.59	-0.15	0.17	0.15

$$\begin{aligned}
 \Delta\mu_u &\equiv \mu_{\Xi_{bbu}^{*0}} + \mu_{\Xi_{ccu}^{*++}} - 2\mu_{\Xi_{bcu}^{*+}} = 0, \\
 \Delta\mu_d &\equiv \mu_{\Xi_{bbd}^{*-}} + \mu_{\Xi_{ccd}^{*+}} - 2\mu_{\Xi_{bcd}^{*0}} = 0, \\
 \Delta\mu_s &\equiv \mu_{\Omega_{bbs}^{*-}} + \mu_{\Omega_{ccs}^{*+}} - 2\mu_{\Omega_{bcs}^{*0}} = 0.
 \end{aligned}
 \tag{24}$$

We observe from Table 3 that all baryons almost satisfy that condition except the results obtained in Refs. [70,81].

As we mentioned beginning of this section, we work in the $m_u = m_d = 0$ limit. The second splittings can be defined,

$$\begin{aligned}
 \Delta\mu_{u-d}^{QQ} &\equiv \mu_{\Xi_{Qu}^{*0}} - \mu_{\Xi_{Qd}^{*0}}, \\
 \Delta\mu_{u-d}^{QQ'} &\equiv \mu_{\Xi_{Qu}^{*+}} - \mu_{\Xi_{Qd}^{*+}},
 \end{aligned}
 \tag{25}$$

which gives the difference occurring as a result of the different charges of u and d quark. The third splittings can also be defined,

$$\begin{aligned}
 \Delta\mu_{d-s}^{QQ} &\equiv \mu_{\Xi_{Qd}^{*0}} - \mu_{\Omega_{Qs}^{*0}}, \\
 \Delta\mu_{d-s}^{QQ'} &\equiv \mu_{\Xi_{Qd}^{*+}} - \mu_{\Omega_{Qs}^{*+}},
 \end{aligned}
 \tag{26}$$

which gives the difference occurring as a result of the different quark masses of d and s . In the second and third splittings, the heavy quark contribution has been canceled out. Therefore, $\Delta\mu_{u-d/d-s}^{cc} = \Delta\mu_{u-d/d-s}^{bb} = \Delta\mu_{u-d/d-s}^{bc}$ is expected. We can see that most results in Table 4 satisfy this relation, except the $\Delta\mu_{d-s}^{cc}$ of this work.

4 Discussion and concluding remarks

In the present paper we have evaluated the MDMs of the spin- $\frac{3}{2}$ DHBs by means of the light-cone QCD sum rule. The MDMs of the DHBs encode key knowledge of their internal structure and shape deformations. Measurement of the MDMs of the spin- $\frac{3}{2}$ DHBs in future experiments can be very helpful understanding the internal structure of these baryons. However, the direct measurement of the MDMs of the spin- $\frac{3}{2}$ DHBs are unlikely in the near future. Therefore, any even non-straightforward approximation of the MDMs of the spin- $\frac{3}{2}$ DHBs could be very helpful. Comparison of our results with the estimation of other theoretical models is presented. As can be seen from the MDM results of the DHBs given in Table 2, the results obtained using different models lead to rather different estimations, which can be used to distinguish these models. Obviously, more studies are needed to understand the current situation.

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